Pennies from Heaven
by Gerald Fitton

Every time it rains it rains
Pennies from Heaven
You'll find your fortune falling
All over town
Be sure that your umbrella
Is upside down

"Pennies from Heaven" 1936
Lyrics by Johnny Burke (1908 - 1964)
Music by Arthur Johnston (1898 - 1954).

This song was written in the days when a verse before the first chorus was an essential part of every song. The verse for this song contains the famous lines: "The best things in life are absolutely free" and goes on to to advise us that if the sky was always blue or if the moon was always new, then we wouldn't appreciate it. "So it was planned that they would vanish now and then." The word "Pennies" is used metaphorically. The lyrics continue by advising us that we trade these 'pennies' (the raindrops) for flowers.

They don't write lyrics like this anymore! Or am I getting old?

The Scenario

In the spring of each year a shop can stock up on either Umbrellas or Parasols but, in this hypothetical scenario, not both.

The long term weather forecast for the summer of that year is either Wet or Sunny. Information about the possible weather for that summer is in the form of two probabilities which, of course, add up to 1.0.

If the summer is a Wet one then the shop will make a good profit on Umbrellas but a poor profit on Parasols. If the summer is Sunny then the shop will make a good profit on Parasols but a poor profit on Umbrellas.

In the spring the shop orders in bulk from its supplier for delivery in summer. In this rather hypothetical scenario the management must make an irrevocable decision in spring to buy either all Umbrellas or all Parasols (it cannot buy some of each) and it is then committed to the consequences of that decision for the remainder of that year.

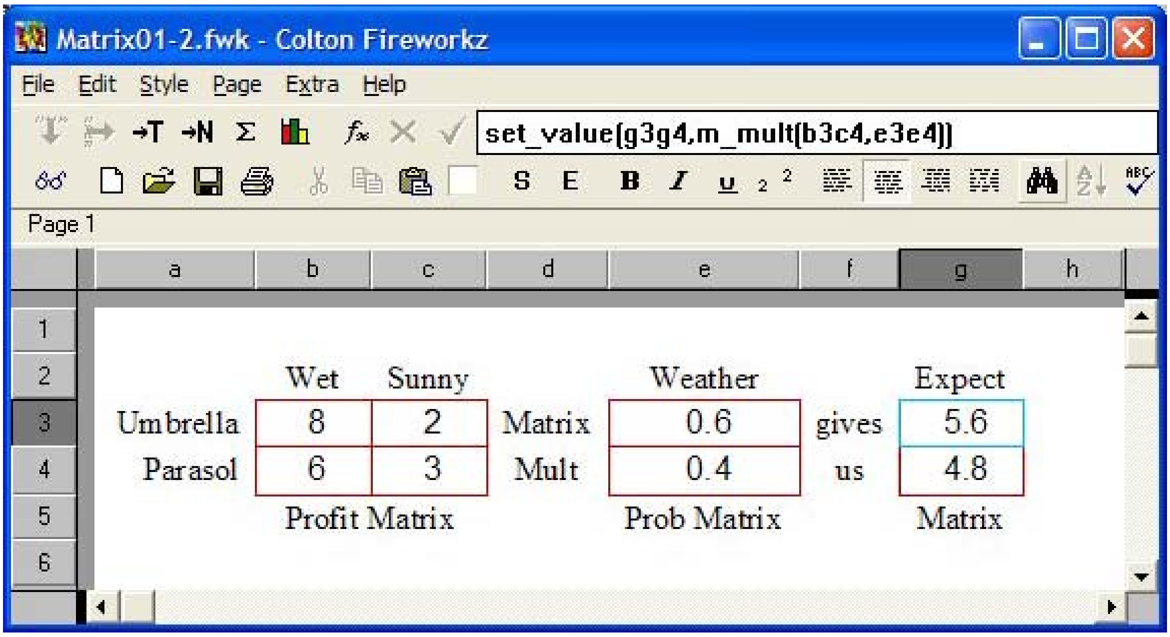
To aid the shop in its decision it uses the long term weather forecast; this is a 'Probability Matrix' consisting of two numbers which add up to 1.0. One of these two probabilities is the probability that the summer will be Wet and the other is the probability that the summer will be Sunny.

The managers of the shop know the profit they will make by buying Umbrellas followed by a Wet summer or a Sunny summer (two numbers). Also they know the profit they will make by buying Parasols if the summer turns out to be Wet or Sunny (two more numbers). These four numbers can be set out in a two way table, the Profit Matrix.

The Calculation

The shop's managers have to decide whether to order Umbrellas or Parasols. In order to do this they call in their consultant (naturally, this is Gerald from Abacus Training) who creates a spreadsheet to do a calculation which will help the managers make this decision.

The calculation built into this expensive but invaluable spreadsheet contains a Matrix Multiplication which (with no extra financial reward) I shall share with you.



If the shop stocks up with Umbrellas and it is Wet then their profit will be 8. Perhaps the units for this 8 is Pounds Sterling (these days called GBP - rather than UKP!) or, because they have to recoup the consultancy fee paid to that expert in Risk Analysis known only as "Gerald", it is more likely that it is millions or even billions of pounds.

You will see this value, 8 (without the units!) appears in cell b3 of the spreadsheet.

In the past I have noticed how quickly Archive readers latch on to systematic layouts, consequently I shall waste no more valuable space in Archive explaining the meanings of the other three numbers which appear in the Profit Matrix of b3c4.

However, it is worth me explaining the meaning of the Probability Matrix of e3e4.

In order to keep it simple, summers can be of only two totally distinct types (I said this was a hypothetical scenario) which I've called Wet and Sunny. The probability of a Wet summer is 0.6 (or 60% if you prefer) and the probability of a Sunny summer is 0.4. Let's hope that in 2009 these probabilities are reversed! I prefer sunny holidays.

My next bit is something which some people find difficult to grasp. I want you to consider all those summers for which the probability of it being Wet is 0.6. Forget about every other type or class of summer. Of these summers 60% will be Wet and 40% will be Sunny. Let's say we have 100 such summers. 60 will be Wet and 40 will be Sunny.

If the shop always stocks up with Umbrellas then, on about 60 out of these 100 summers the shop will make a profit of 8. This totals to 8 x 60 = 480 in whatever units the profit is measured. Also, on the other 40 Wet summers, the profit will be 2 making 2 x 40 = 80.

The total profit for the 100 summers is (approximately) 480 + 80 = 560 or, if you prefer, an average profit of 5.6 units per year over the 100 years.

Now let's see if you've got it! If, during these 100 years, the shop always stocks up with Parasols then the average yearly profit will be (6*0.6) + (3*0.4) = 4.8 units.

The conclusion is that, over the long term, when the weather forecast is 60:40 Wet:Sunny then the shop should stock up on Umbrellas all the time because, by doing that, it will average (5.6 - 4.8) = 0.8 more profit than if it stocked up on Parasols.

Decision Making

There are many criteria used for making decisions like this. The method I have described is called "Maximising the Expected Value of the Decision" or "The EV Method" for short!

When using this EV method it is important that the shop has enough working capital (including an overdraft facility) so that it can ride out the bad years and continue to be in existence when the good years come back! It is all too easy to forget this and treat the EV method of decision making as the only 'right' criterion. Seeking maximum profit without considering the consequences of having 'a bad year' (or two) is the sort of thing which manifests itself in spectacular growth followed by an equally spectacular collapse.

This warning about using EV exclusively is embodied in Gerald's Second Rule of Gambling, "Never gamble what you can't afford to lose".

The Spreadsheet

For this simple, totally hypothetical and devised scenario, there is only one formula in this spreadsheet; it is this one formula for which our honoured, knowledgeable and greatly renowned consultant (known only as "Gerald") has been paid his undoubtedly deserved fee.

That formula appears in cell g3. It is: set_value(g3g4,m_mult(b3c4,e3e4)).

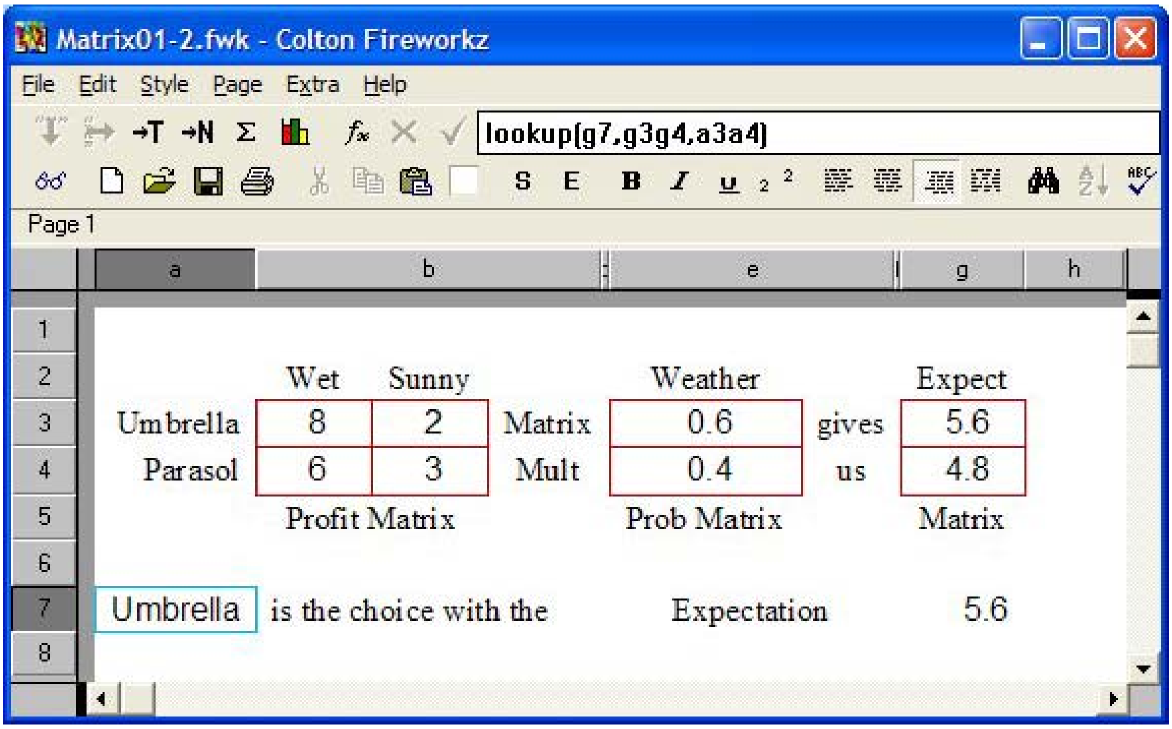
The set_value(destination, source) function expands the result of the matrix multiplication into the two cells g3 and g4. The value returned in g3 is the 'Expectation' (think of it as the expected annual profit averaged over a lot of years) if the Decision is to buy Umbrellas. The value in g4 is the Expectation if the Decision is to buy in Parasols.

The function m_mult(matrix1,matrix2) executes a matrix multiplication of the matrix 'matrix1' and 'matrix2'. The value in g3 is (8*0.6) + (2*0.4) = 5.6. The number of columns in the first matrix must be identical to the number of rows in the second matrix. If they are not the same then an error message is returned.

In this hypothetical, both the Profit Matrix and the Probability Matrix are 'given', known values. They are used to calculate the Expected Value Matrix using matrix multiplication.

Finding the Maximum Expectation

Look at the next screenshot and you will see that I have extended the spreadsheet by adding row 7. Although this row is rather superfluous for this simple, hypothetical scenario, it will make it much easier to identify the 'best' decision (based on maximising the EV) when there many more possible choices each returning its own value of the Expectation.



The formula in g7 is max(g3g4). This function returns the highest value in the Expectation Matrix. The formula in a7 uses lookup(key,lookup-range,output-range). You will see the formula line contains the formula lookup(g7,g3g4,a3a4). In this case the maximum value in the range g3g4 is 5.6 in g3. The lookup formula searches the range g3g4 for this maximum value and then, having found the row in which 5.6 occurs, it returns the corresponding value from the range a3a4.

This extended spreadsheet in this case returns the value "Umbrella".

Not so hypothetical

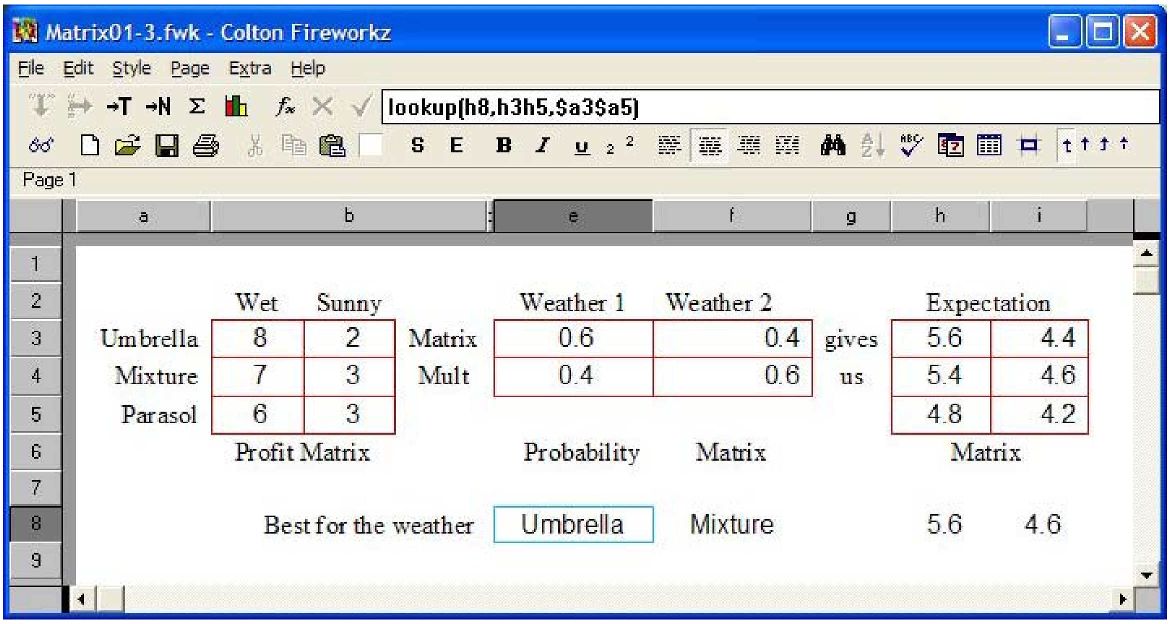
I have chosen as my example for calculating the expected value of a range of alternative possible decisions something which is so trivial that it will appear almost ridiculous. Nevertheless, it shows all the features of this very powerful method for (automatically) making decisions which will maximise the long term profit (provided that you can get through the 'hard times') when the risks of success and failure can be estimated.

I am sure that you will be able to invent for yourself much more complex scenarios. I shall consider a simple modification to the spreadsheet by allowing the shop to buy a particular mixture of some Umbrellas and some Parasols rather than all Umbrellas or all Parasols.

When there are a very large number of possible decisions (many rows in the Profit Matrix) then this method will still return the answer returning the best Expectation.

In a similar manner the Probability Matrix can be extended to many columns, for example, to cover different possible weather forecasts or weather forecasts made using different (perhaps more costly) resources. In this case the Expectation Matrix is not one column but many columns, each column corresponding to a different means of forecasting the weather.

An example



In the example shown in this screenshot the decision to be made is between three alternatives for two different weather forecasts. Both the Profit Matrix and the Expectation Matrix have two columns, one for each weather forecast.

If the weather forecast is “Weather 2” then the maximum expectation is 4.6 (in cell i4) and this is achieved by choosing the “Mixture” of Umbrellas and Parasols represented by row 4 in the Profit Matrix.

In cell e8 have entered the formula lookup(h8,h3h5,$a3$a5). I have replicated this formula into cell f8. The “$a” in this formula ensures that the value returned is always from column ‘a’ which contains the name of the decision, “Umbrella”, “Mixture” or “Parasol”.

This formula returns the value “Umbrella” if the weather forecast is “Weather 1” but “Mixture” if the forecast changes to “Weather 2”.

Summary

This article is not about using your umbrella or parasols to catch “pennies from heaven” but it does demonstrate how the function ‘m_multiply(matrix1,matrix2) can simplify the construction of a spreadsheet used to find an Expectation Matrix in a complex scenario. A spreadsheet can do a lot of ‘what if’ sums for you very quickly.

What it can not do is make the Decision for you. You should never trust a computer to make a decision for you unless you know exactly the limitations of the decision making process used by the computer program - even if the spreadsheet has been constructed by that expert, “Gerald”.

Bio-bit

The quotation, “There are lies, damned lies and statistics” implies that the misuse of Statistics is a sophisticated form of telling.lies! Gerald maintains that Statistics, honestly applied can be not only illuminating - but, more than that, they can be Fun!

Bio-pic

Box out - When the Probability is Close to Zero

I must repeat my warning about using Expected Values as the sole decision making tool for situations in which the outcome is uncertain. My warning can be embodied in Gerald’s Second Law of Gambling: “Never bet what you can’t afford to lose.”

Gerald’s Second Law comes into its own when the probability of ‘success’ is close to 0.0.

In an earlier article I have said that, “There is no profit without risk”. I have emphasised this message with Gerald’s First Law, “Never bet on certainties”. One man’s certain winner is another man’s certain looser! Probabilities are not absolute but depend on the amount of information known to the person estimating the probability.

“If the probability of success looks to be too good to be true - then it probably is!”

An example of a situation in which the probability of winning is close to zero is buying a ticket in the national lottery. Another is paying home insurance against the possibility of something disastrous happening to your house. In this second scenario paying your insurance premium is making a bet. Somewhat perversely, you achieve 'success' in this gamble if you 'win' the compensation because your house has been destroyed!

In either of these situations you are making a bet for which 'Maximising the Expectation' would lead you to the decision, "Don't make the bet!".

There is no doubt that winning the lottery could change your life but the probability of success is minuscule. The only redeeming feature in favour of making this bet is that you can afford to lose your stake of £1. "The house always wins."

When it comes to house insurance then there is no doubt that the premium asked by the insurance company is not what, in the jargon, is called a 'fair bet'. In the long run the insurance company, like the 'house' will always win these gambles.

If you do not insure your house then you are still gambling but your 'stake' is no longer the insurance premium but is the value of your house! If you made your decision on the basis of 'Max EV' then you would decide not to pay the premium but take the risk yourself (in house). However, if you lose this bet then the consequences for you will be disastrous.

The correct decision for an individual to make is to pay their insurance premium even though they know that the bet is not a 'fair bet'. The situation might be different for a large corporation. Indeed, you, as an individual, want the insurance company to charge more than a 'fair bet' because, that way, you will know that the probability of the insurance company being in existence and having the funds to pay you when your bet is 'successful' (your house has burned down) is much higher than if the company is charging too little to remain viable in the long term.

So I repeat:

"Never bet what you can't afford to lose."

... particularly when the probability is close to zero