

# That's a Good Question

## by Gerald Fitton

*No man is an Iland ...  
... because I am involved in Mankinde,*

*Devotions Upon Emergent Occasions, Meditation 17  
Written in 1623 by John Donne (1573-1631).*

Although it is just about possible to define a number (such as 'one') without reference to any other number, it's really hard going. Whatever the philosophical questions which can and are asked about 'one', I am firmly of the opinion that Arithmetic only gets going when we put a bunch of numbers together and start looking at the relationships between them. Indeed, as I have asked before, what would 'one' be without 'two'?

Many of you will know of the 'chat list' called "42". It was called that because "42" is the answer to the Ultimate Question about "Life the Universe and Everything" returned by the computer, Deep Thought, created by Douglas Adams (in his Sci-Fi book lovingly referred to as 'H2G2' by his devoted fans); on that chat list it is possible to discuss almost anything!

I have spent time pondering that Ultimate Question and the answer I have come up with is "Relationships". If there is one thing which gives our life meaning and relevance that thing involves our relationships whether these are the relationships we have with our family, those we form with our friends and colleagues, or even the relationship we have with the god of our choosing. The 'Meaning of Life' lies with the relationships we form.

Mathematics is all about relationships; it is about relationships between numbers, between unknowns such as 'x' and 'y', between lines and points, between derivatives and integrals and between many other concepts which mathematicians love to discover or invent.

In Meditation 17, John Donne reflects on the relationship we have with every other human being. We depend on each other; we depend, we need to depend on the talents of others.

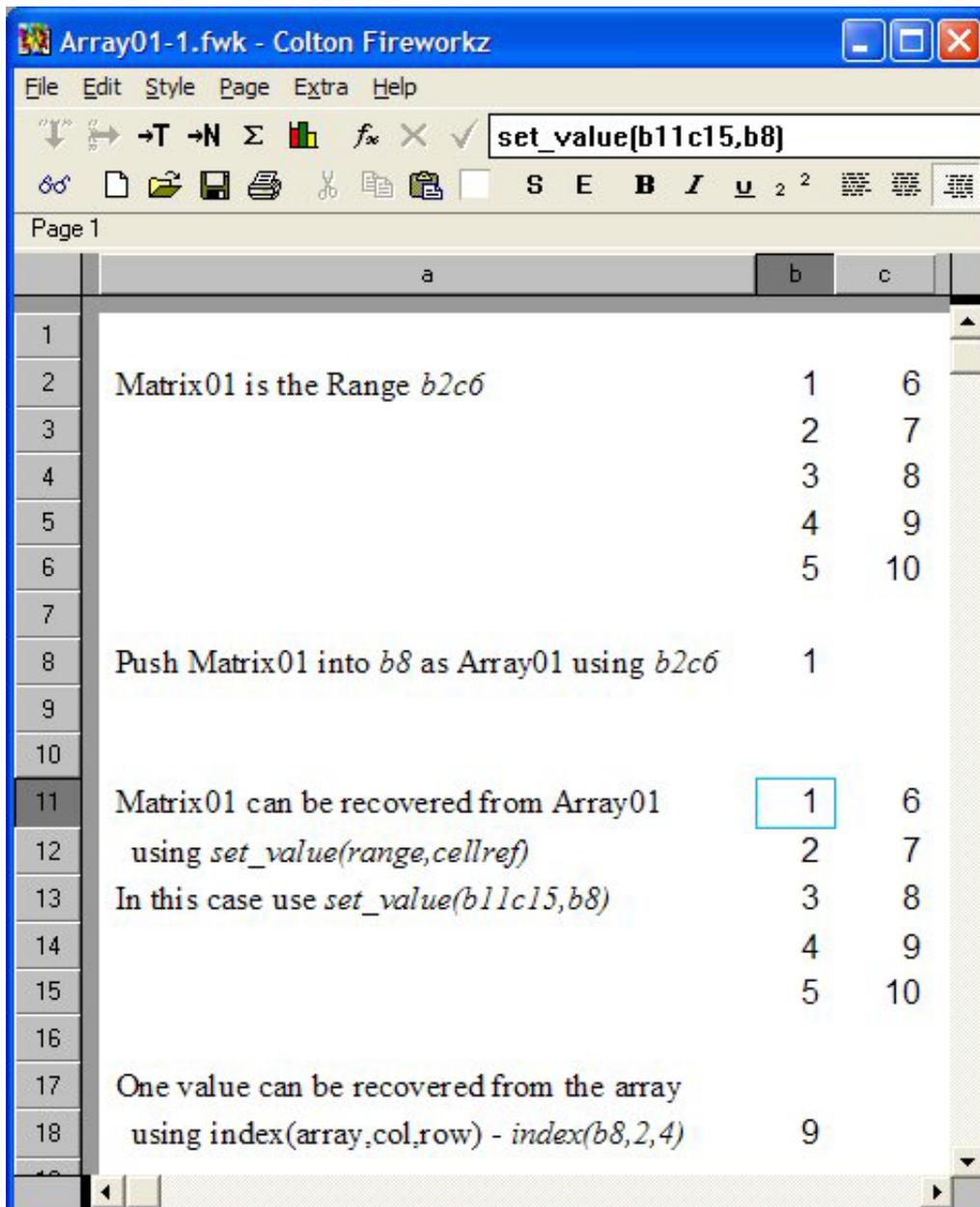
When I first conceived this article I intended to reflect in great depth on those collections of numbers which Mathematicians call Arrays and Matrices. However, when I started putting my article together in earnest I found that a full explanation of Arrays and Matrices cannot be crammed into part of one article - it needs more space than that.

Also, this month I have to complete my story about the students who took the Numeracy test. You might be surprised by what I shall tell you about the 'strong' students - read on.

So this month I shall make a start on Arrays (consider it a taster) leaving me with enough space to complete the story which I started last month. I shall use Fireworkz as my choice of spreadsheet. Pipedream 4 (but not PipeDream 3) contains all the features I shall use when demonstrating the features Arrays and Matrices. In addition to Fireworkz, there are many other RISC OS, Windows and Mac spreadsheets which support Arrays and Matrices.

### **What are Arrays?**

Let's start with a 'Range'. For my screenshots I am using Fireworkz for Windows!



The set of numbers in cells b2 to c10 is called a Range; in Pipedream and Fireworkz a range of cells can be written as b2c10. Some spreadsheets insert a colon between the b2 and c10 so that the range would be written as b2:c10.

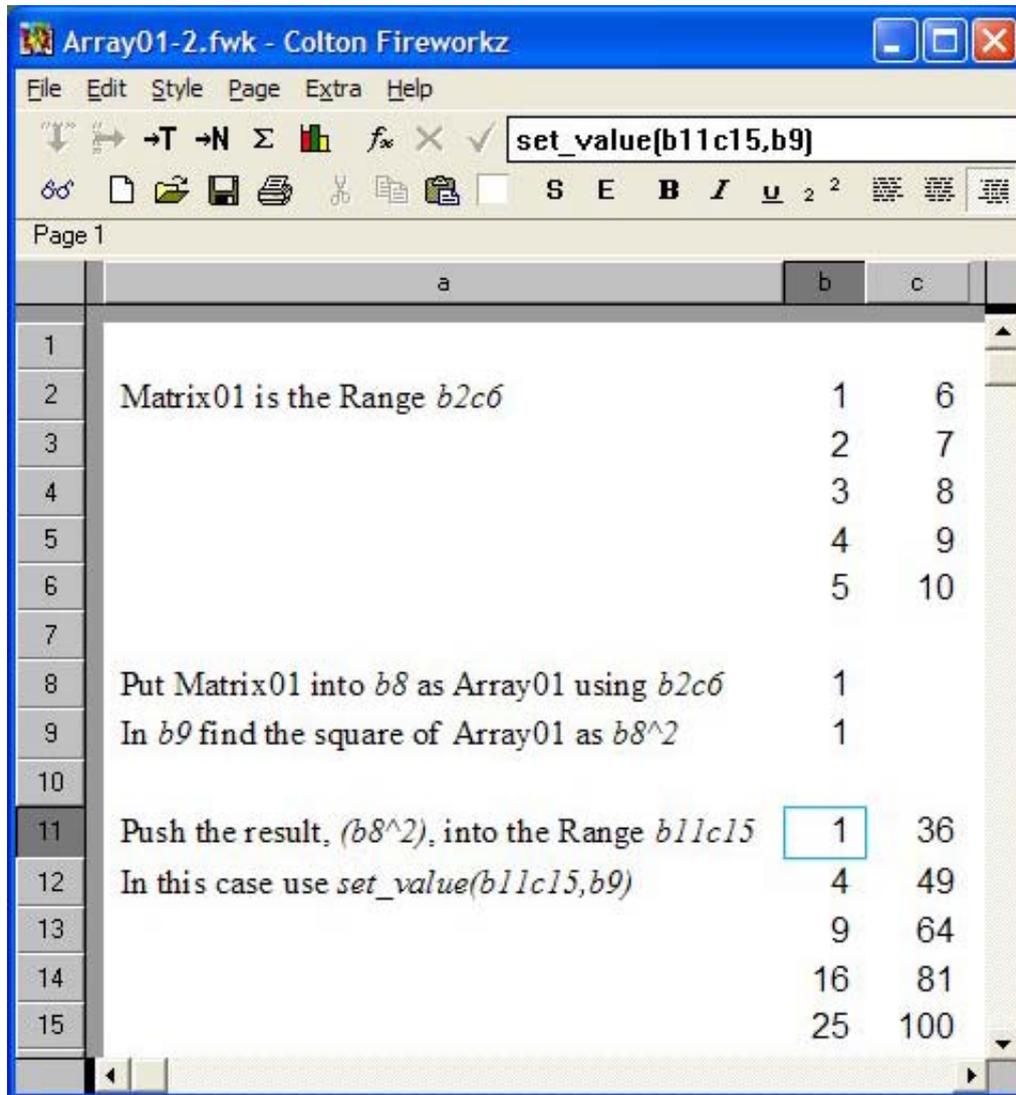
Now look at cell b8. It contains an Array. You wouldn't know it just by looking at the value, "1" displayed in b8, but this one cell contains all the ten numbers of the Range b2c6 as an Array two columns wide and five rows deep.

The very simple formula in cell b8 is b2c6. I assure you that all ten numbers from the Range b2c6 are safe and sound within b8. I'll try to prove I to you!

In cell b11 I have used the function `set_value(destination,source)` with the cell b8 as the source and the Range b11c15 as the destination. What this function does is to expand the contents of the cell b8 into the Range b11c15. We are back to where we started.

## Multiplying Arrays

Now look at the next screenshot.



The screenshot shows a spreadsheet window titled "Array01-2.fwk - Colton Fireworkz". The formula bar contains the text `set_value(b11c15,b9)`. The spreadsheet has three columns labeled 'a', 'b', and 'c'. The data is as follows:

	a	b	c
1			
2	Matrix01 is the Range <i>b2c6</i>	1	6
3		2	7
4		3	8
5		4	9
6		5	10
7			
8	Put Matrix01 into <i>b8</i> as Array01 using <i>b2c6</i>	1	
9	In <i>b9</i> find the square of Array01 as <i>b8^2</i>	1	
10			
11	Push the result, ( <i>b8^2</i> ), into the Range <i>b11c15</i>	1	36
12	In this case use <i>set_value(b11c15,b9)</i>	4	49
13		9	64
14		16	81
15		25	100

The cell b9 contains the simple formula  $b8^2$  (b8 squared).

Array multiplication is different from matrix multiplication. I gave an example of matrix multiplication last month and there is another one later in this article.

When multiplying two arrays together using the formula ( $array1*array2$ ), numbers in the first array are multiplied by the corresponding number in the second array; the result of the multiplication is a third array of the same shape and size. The array in b9 contains a set of ten numbers; each number is the square of the number in the corresponding position of the array in b8. By-the-way ( $array1*array2$ )=( $array2*array1$ ); we can change the order.

Look at the numbers in the range b11c15 and you will see that each number is the square of the number which is in a corresponding position in the range b2c6. Array multiplication is an easy way of finding the square, cube or even the cube root of a set of numbers.

## Multiplying Matrices

It is my intention to spend more time on Matrices in another article so, for this month, here is just one example of matrix multiplication. Look at the screenshot.

The screenshot shows a spreadsheet window titled "Matrix01-1.fwk - Colton Fireworkz". The formula bar contains the formula `set_value(f6i7,m_mult(b6d7,f2i4))`. The spreadsheet grid shows the following matrices:

Matrix M (2x3):

1	2	3
2	4	6

Matrix N (3x4):

7	8	9	10
11	12	13	14
15	16	17	18

Resulting Matrix M\*N (2x4):

74	80	86	92
148	160	172	184

The size of the matrix M is 2 rows by 3 columns (2X3) and N is 3X4. The Fireworkz formula for matrix multiplication is `m_mult(array1, array2)`; this formula will return M\*N as a 2X4 array contained within a single cell. I have expanded the contents of the cell into the range f6i7 using `set_value(destination,source)`. The formula in f6 is a combination of these two functions, the matrix multiplication and its expansion into the range f6i7.

The value returned in i6 is  $(b6*i2 + c6*i3 + d6*i4)$ ; now see if you can follow this pattern to calculate the other 7 components of M\*N.

Array multiplication requires the two arrays to be of identical size; matrix multiplication requires that the first matrix has the same number of columns as the second matrix has rows. In array multiplication corresponding elements are multiplied together; matrix multiplication uses a completely different procedure, the procedure I have shown here.

I shall return to matrix operations in another article another day!

## To Many Hard/Easy Questions

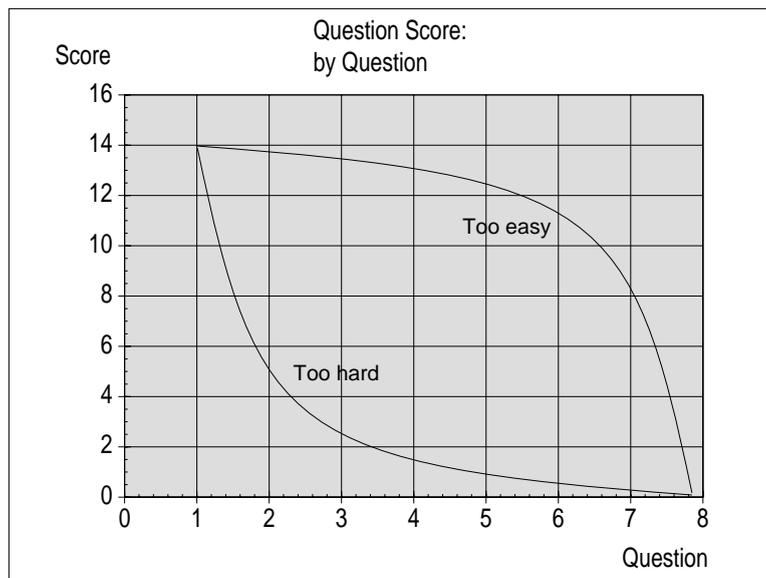
Having made a start on matrices I shall leave this subject in order to complete the story of my students. I wonder how many of you will guess what happened to the strong students?

I bet that you will be surprised - so, if you wish to be surprised, then read on.

One of the problems of writing a test paper is that you mustn't include too many easy questions nor too many hard ones.

Ideally (in this case) what we need is a set of questions which will discriminate well at all ability levels. The line graph below demonstrates the sort of Score/Question graph which you get if you have too many easy or too many hard questions.

If you have many (too many?) hard questions then you may be able to identify the subtle distinctions between the strong and strongest students to a high degree of accuracy; Mensa set their tests this way to identify with great accuracy the 2% who have the highest IQ; Mensa don't really care about the scores of those with low IQs. If you include only hard questions then you will not have data which discriminates well between weak students.



To include too many easy questions is an equally bad mistake. The ideal Question/Score graph for my purpose is one which is a fairly straight line from upper left to lower right.

The technique I actually used to fine tune the Numeracy test paper was only partly as I have described above. The fundamentals of the method are as I indicated; what I did in addition was to fine tune the distractors so that they were all equally distracting.

All the analysis on both the Numeracy and the Literacy questions was done by me at home using PipeDream initially on my A440 and later on my RiscPC.

After a while I managed to get a feel for what made good questions 'good' and what made good distractors 'good distractors'. I developed a set of general principles which the Literacy team found that they could apply to their questions.

### What were we testing?

After a year or so, I had a set of questions which produced consistent results. There were no bad questions - though some were more reliable as discriminators than others.

The Numeracy papers contained mainly abstract simple arithmetic. The paper started with the addition of one and then two digit integers, working up through subtraction without the need to 'borrow', then to three digit addition with carry and subtraction with borrowing.

In the next section there were questions which involved proportion, ratio and percentages. Following that section values had to be read off simple graphs. Another section involved approximations. In this section the multi-choice answers never included the exact answer to the sum so the student had to find the nearest answer.

Finally I included concrete examples which included distance, area, volume, money, and compound measures such as speed (m per sec) and prices (p per kg). I did not test numbers larger than 999, nor negative numbers, nor fractions other than  $\frac{1}{4}$ ,  $\frac{1}{2}$  and  $\frac{3}{4}$ .

I found that questions including pi (area or circumference of circles) were unreliable indicators, fractions other than those mentioned above were unreliable indicators, sums involving more than four digits were unreliable as were many other things I won't bore you with. By unreliable I mean that the Score/Question graph showed many outliers.

I put this down to the emphasis given to teaching these things at the various schools in our area. For example some schools taught the uses of pi and others didn't. What I wanted to test was whether the students had been able to learn, remember and use the things they had definitely been taught. If they hadn't been taught about pi then they wouldn't be able to answer a question which needed knowledge of its properties however 'clever' they were.

So what were we testing?

One thing which nagged away at the back of my mind for a long time was the high correlation with the Literacy test and my insistence that the Literacy test should be tailored so that it yielded results which correlated well with the Numeracy test.

Did I change the Literacy test to prove my hypothesis? Had I been unfair to the new Lecturer? I still don't know the answer to these questions.

### **What did we do with the results?**

One thing we did was to offer extra lessons to those with low scores.

The take up of the extra lessons was about 50%, some refused to join; those who took them improved rapidly under the guidance of sympathetic teachers with low class sizes.

In the second half of the second year I ran what might be called a 'progress' or 'exit' test containing similar questions which confirmed the opinions of these specialist teachers.

Over the two years, those who had attended the extra lessons improved their score considerably; many obtained Distinctions in their main core subjects.

It is my opinion that these students realised how hard work could improve their performance and they became highly motivated to do as well as they were able.

## **The High Scorers**

In 1985 (or thereabouts) I discovered something else from the screening test results. It came as such a shock to me that I didn't believe it was due to anything but chance for another two years. I looked at the screening test scores and I identified the top 15% of students (about 20 to 25 students) and checked on their achievement with us after the two years they'd been with us.

To my utter astonishment (and yours?) most of these 'strong' students never finished the course. Two students completing the course (out of the top 15% of 150) was the highest we ever achieved in the first four years of running this test. More usually it was zero. Furthermore, whilst those students were with us they were all sorts of trouble ranging from skipping lessons, handing in work late or not at all, to being disruptive in class.

At first I disbelieved the evidence telling myself that it was 'just a random glitch', then I spent another year trying to discover what was going wrong. I think I might have got somewhere because all but one of the high scoring students who completed were members of my tutor group and they had massive trouble with the Lecturers.

I tried to get the Educational Psychologist (he was a Part Time consultant) interested in my findings. His (free of charge) contribution was that it was all down to Student Motivation. Unfortunately the College wouldn't pay him to investigate my findings in greater detail so I never got a real expert to look into it.

## **Boring**

You will appreciate that once students have left college it is difficult to find out why they left if only because they are no longer around to ask. What I did was to identify these students from the screening test and then, well before they left, I would ask them what they thought about the course, etc. One common complaint was that their (often original) work was marked down for petty reasons "just because it isn't what the Lecturer said". Another much more common comment was that they were "Bored".

Like many words, the meaning of "Bored" has changed with time. When a student said, "Bored" it didn't exactly mean that they found what they were doing uninteresting. The modern usage by students, and maybe other young people, of "Bored" implies a lack of understanding of what is important to the student by those in authority - in management jargon there is a 'communications gap'.

## **Later**

Later, as a result of one of my recommendations we merged the Numeracy and Literacy tests mixing the questions. In addition we expanded it to include behavioural and motivational questions and other non academic questions such as sporting or leisure interests (to hide the true reason for the test) and then I analysed the data in different ways for different purposes by selecting only those questions which were relevant to whatever we wanted to test.

## Summary

This month I have given you a 'taster' of Ranges, Arrays and Matrices. At sometime in the future (perhaps not next month) I shall expand on this brief introduction.

We never solved the problem of retaining the strong students. Personally, with my own tutor groups, I did very well (the only ones I 'lost' were promoted to a higher level course) but I was the exception rather than the rule. Even now that I am retired and have the leisure time to think about it, I do not know what we (meaning I) could have done - but I do regret my failure. I am sure that many of you reading this will have been rejected by formal education for 'thinking outside the box'. So what do you think is the solution?

## Bio-bit

As an Engineer Gerald made things - that worked. As a Company Director he made profits - mainly for other people. As a College Lecturer he claims he 'made people' - become more fulfilled than otherwise they might have been. Of his three careers Gerald regards his time at College as the most rewarding.

## Bio-pic



## Box out - Humanism

The quote at the beginning of my article is written in 'Olde English'; in more modern English it reads:

*No man is an island ...  
... because I am involved in mankind ...*

John Donne was appointed Dean of St Paul's in 1621 and held this well paid position until his death in 1631. It was in 1623 at the age of 51 that he wrote his 23 Devotions. The lines I have quoted are from his 17th Devotion. It asserts that we, as human beings, have an unbreakable symbiotic relationship with all mankind.

I interpret this to mean that it is incumbent upon everyone of us to use our talents, which some would say are God given, for the benefit not only of ourselves but of all mankind.

Capable teachers will feel an inner need to use their talents to enhance the lives of all those whom they teach, even the 'wayward' ones. The compassion and tolerance of carers never fails to impress me. However, beyond these more obvious talents, the skilled fitter repairing my central heating system has a talent which it is his 'Duty to Humanity' (and therefore to himself) to practice to the best of his ability. When we sweep our floors, wash our dishes, drive our cars, even when we write an article for Archive (!), we must resolve to do the best job of which we are capable. Each of us should do this because we know that we have a bounden 'Duty to Humanity' to place our talents at the service of Mankind.

Recently, in March 2009, the Archbishop of Canterbury, Dr Rowan Williams, spoke about the way in which we, Mankind, are conspiring to bring about "runaway climate change". He said, "... to suggest that God might intervene to protect us from the corporate folly of our practices is as unchristian and as unbiblical as to suggest that He protects us from the results of our individual folly or sin ... (God) will not (provide) a safety net that guarantees a happy ending in this world".

Whether or not we believe in God we still need to accept that we have responsibilities not only to those who are living now but to all future generations. otherwise, to quote the Archbishop, we (Mankind) will be "choked, drowned, starved by (our) own stupidity".

Both the Humanist and the Christian have common cause to regard themselves as having responsibilities to the whole of Mankind, both present and future.

This section of John Donne's 17th Devotion ends with the famous lines:

*And therefore never send to know for whom the bell tolls:  
it tolls for thee.*

These lines reinforce the view that we're all in this world together and, if we decide that we will not help each other, then all of us will be "diminished" by our selfishness.

Gerald