Gerald's Column
by Gerald Fitton

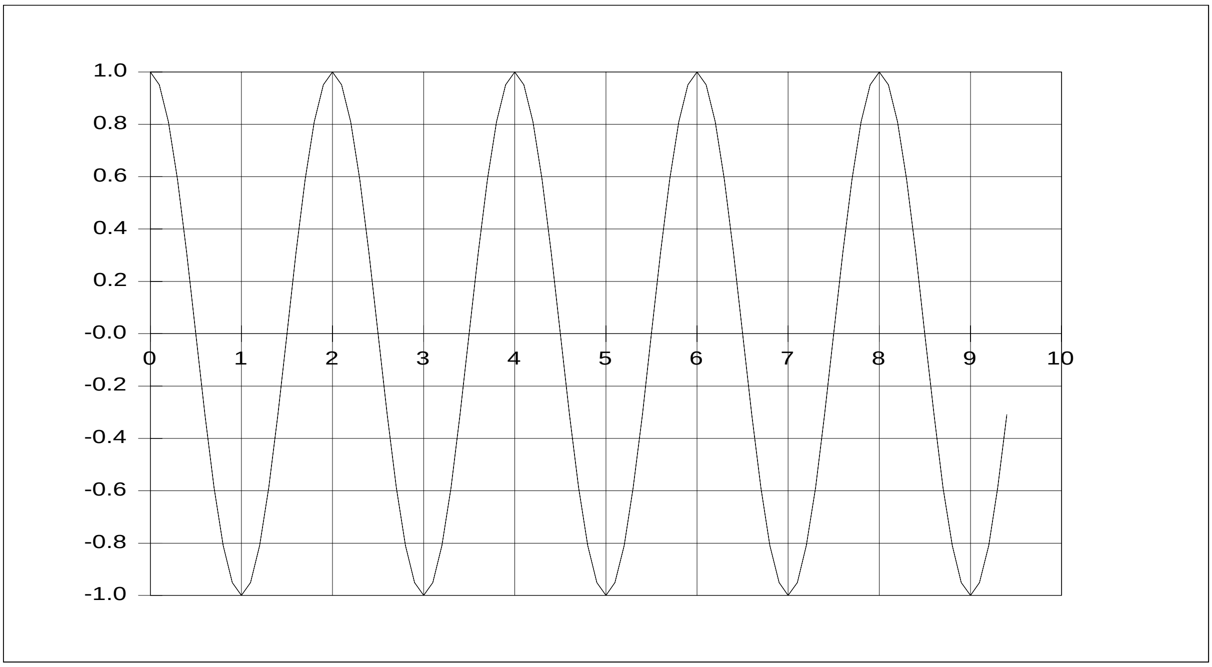
Am I alone in hating speed bumps? It's not the fact that I have to slow down for them that troubles me but the shock to the suspension of my otherwise beautifully smooth riding car when I come back to earth that jars my sensibilities! I shall not use the rising part of a speed bump for my article. I shall assume we are moving along a level road which suddenly drops away. This scenario is shown in the storyboard below.

The first three frames of this storyboard illustrate a dynamic situation which is not too unpleasant. It is the final frame and what comes immediately afterwards which upsets my sensibilities. My dear little car (particularly the suspension) likes it even less.

In this article I shall explain to you, with a graph or two, the oscillations experienced by my car, my passenger and last but not least my tummy, immediately following frame four.

Oscillation

After frame four most of my car (not the wheels - I hope) and I oscillate up and down relative to the (assumed horizontal) road. In the graph below, time runs from left to right and a fixed point in the car (my tummy!) starts (when time = 0) at y = 1. The road drops away through a height of one unit. Of course the wheels drop to the new lower level very quickly and make firm contact with the road. They need to do this so that the car can and does retain traction and allow me to control the steering. However, inside the car my tummy follows the line of the the graph or, more accurately, it would do if it were not for something clever built into the suspension called 'damping'.

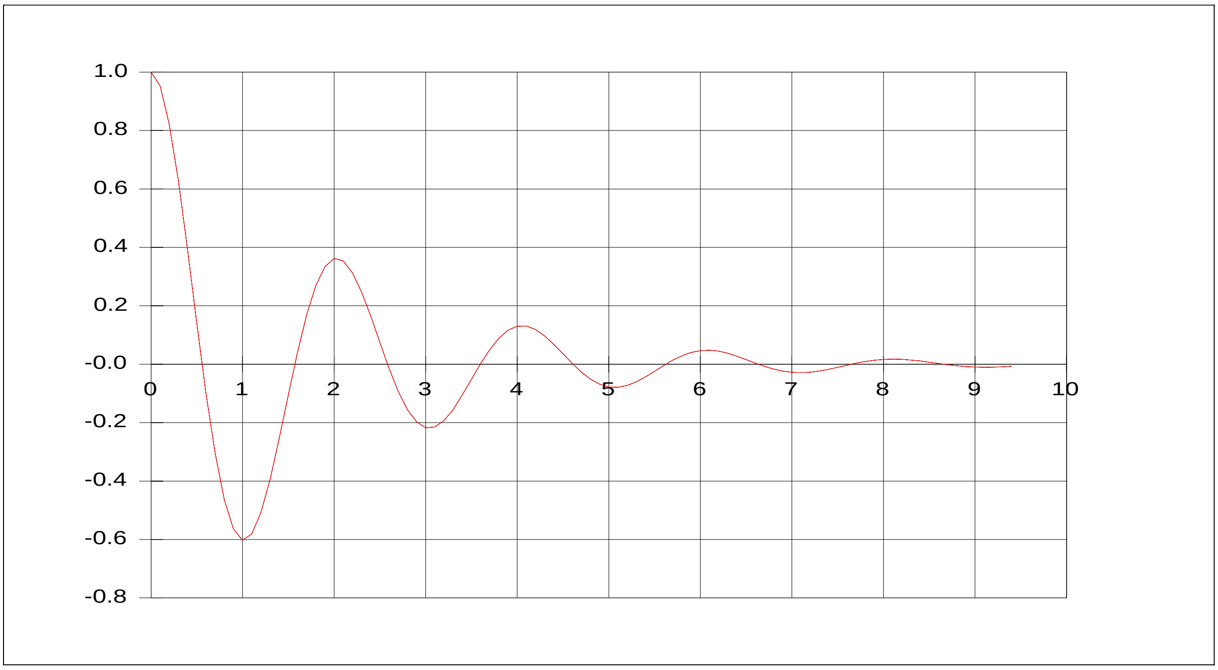


Damping

Fortunately for my passenger (who has a much weaker stomach than I and doesn't like the high 'g' forces which occur at the times when y = 1) my suspension is not frictionless. Indeed, unlike my my first car, a 1936 Riley 9 with leaf springs and a torsion bar, the car I have now has a wonderful system. It has hydraulic dampers which prevent my passenger experiencing a nasty attack of motion sickness resulting from such undamped oscillations.

Instead of my tummy (and that of my passenger) oscillating forever (see above) the amplitude dies away and I reach an equilibrium at y = 0. The graph below shows the sort of thing which actually happens. The energy in the oscillation is converted into heat. Of course this heat warms up the air around the car. So far as getting from A to B is concerned speed bumps are a waste of fuel! Fuel is used to create the oscillation which the dampers convert into heat - contributing to global warming! I don't like speed bumps!

I have coloured the damped line red for 'hot'!



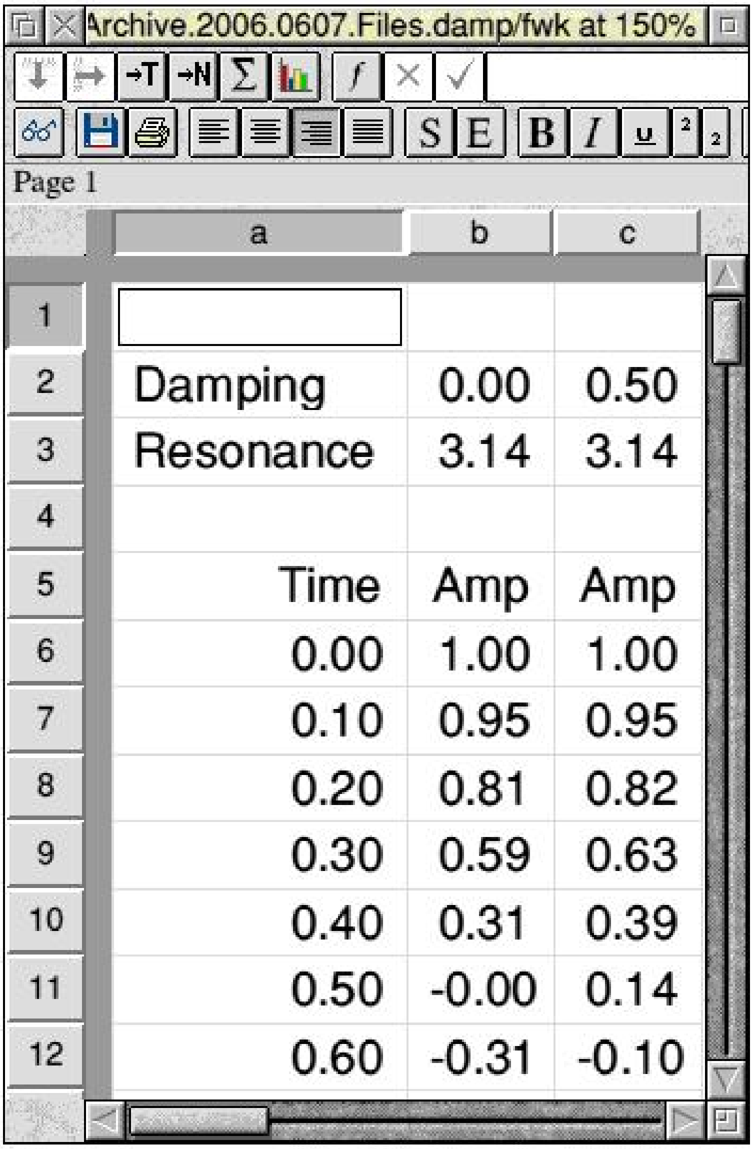
If the suspension of your car is in good working order then the oscillation will stop in fewer than the four cycles shown in the graph.

If your springs are weak or if the bump in the road is too much for your suspension then your car may 'bottom out' on the shock absorbers. The mathematical model I'm using doesn't include the possibility of 'bottoming out'. Bottoming out isn't a good thing for your car. You're probably going too fast so slow down!

A fault I had on one brand new Ford (it must have been a 'Friday Car', there were so many different things wrong with it) was that one of the hydraulic dampers had leaked (all the oil inside it was gone). It was no longer effective. On that wheel the oscillations continued for a long time, more than four cycles, giving a very strange feel to the car. A low value for the damping coefficient represents the condition of this wheel on my Ford.

Resonance and Damping

The screenshot below shows part of the Fireworkz document which I have used for generating the graphs shown above. Column 'a' contains the 't' values ("Time") in intervals of 0.10. Columns 'b' and 'c' contain the y values (the amplitude, "Amp" of the graph) for the undamped (first graph) and damped (second graph) respectively.



	a	b	c
1			
2	Damping	0.00	0.50
3	Resonance	3.14	3.14
4			
5	Time	Amp	Amp
6	0.00	1.00	1.00
7	0.10	0.95	0.95
8	0.20	0.81	0.82
9	0.30	0.59	0.63
10	0.40	0.31	0.39
11	0.50	-0.00	0.14
12	0.60	-0.31	-0.10

The y values are calculated using a rather imposing formula having the two parameters shown in row 2. These parameters are usually given the names "Damping" (for which I use the letter 'k') and "Resonance" (for which I use the letter 'r').

The formula is best written in terms of a frequency of oscillation which is slightly slower than the resonant frequency of the system. This lower frequency (which I shall call 'f') is given by the formula f = sqr(r^2 - k^2). Because of the square root in the formula, 'k' (the damping) can not be larger than 'r' (the resonance) if you want to do calculations using 'real' (rather than complex) numbers. The formula still works if your software can do calculations using complex numbers! Most spreadsheets can't do complex numbers easily.

Fireworkz does have a bank of complex functions (including trig and exponential complex functions) so it can handle complex number calculations. However, for this application I shall keep it simple; I shall leave the use of Fireworkz with complex numbers for another day. What we'll do today is to use 'real' functions of 'real' numbers to plot 'real' graphs.

The formula I have used is:

y = exp(-k * t) * [cos(f * t) + (k/f) * sin(f * t)]

where t is the time which has elapsed since dropping off the 'hump'.

The part of the formula within the square brackets represents the undamped oscillation with the initial condition that dy/dt = 0 (I take off horizontally); it has the frequency f which you will remember is slightly lower than r. The exponential term, exp(-k * t), causes the oscillations to gradually die out; that is why 'k' is called the damping factor.

Custom Function

One very useful (when you know how) feature of Fireworkz is that you can write your own functions. These are called Custom Functions. Conventionally (not essentially) custom functions have file names starting "c_...". The one I have written to calculate y using the formula above is called [c_damp].

Writing custom functions using Fireworkz is not difficult but I must not allow myself to be sidetracked with my car in mid air (so to speak) so we'll leave custom functions as something to look forward to at sometime in the future.

Resonance

In order to make the graph look 'clean and neat' I have chosen a resonant frequency of PI (roughly 3.14159...) because that makes a complete cycle two units. Have a look at the first graph and you'll see what I mean. Maximum 'g' is experienced when 't' is an integer.

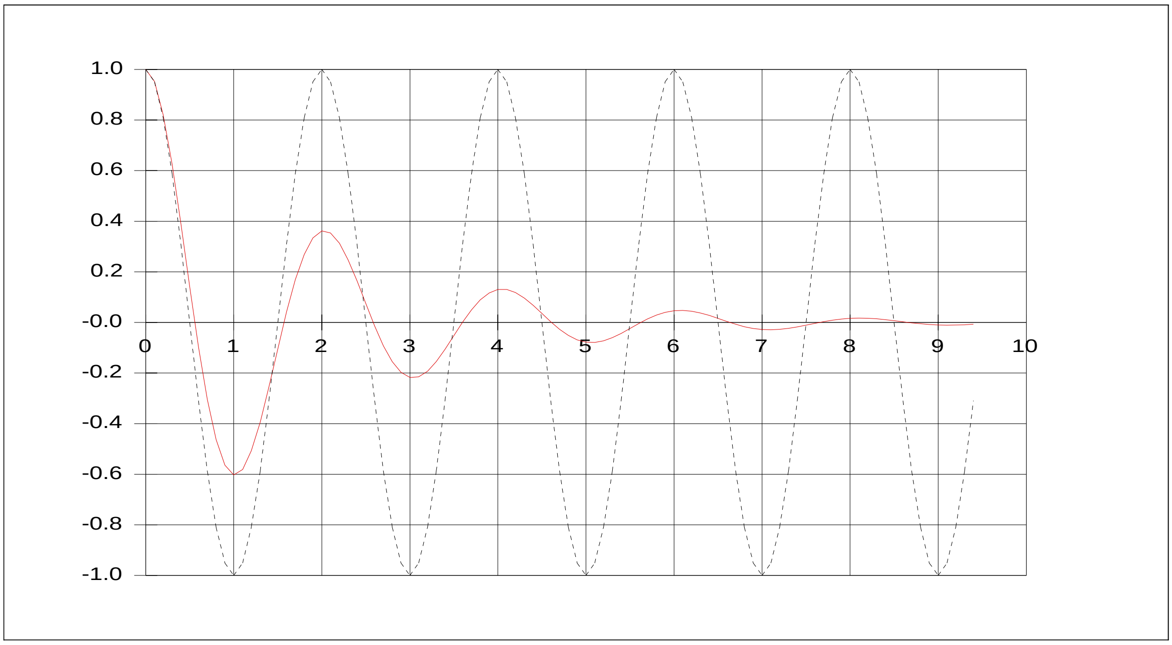
Undamped Oscillations

Giving 'k', the damping coefficient, a value of zero produces the first graph of this article. The frequency of oscillation is the resonant frequency and the amplitude of the oscillation does not diminish with time.

Damped Oscillations

If you look at cell 'c2' of the spreadsheet you will see that my second graph has been created using a damping coefficient of k = 0.5. Because of the limitations of formula using 'real' numbers you can not enter a value of k larger than PI. Using k = 0.5 you will see that there are four good cycles before the amplitude has decayed to something which might be regarded as negligible (compared with the uneven surface of the road).

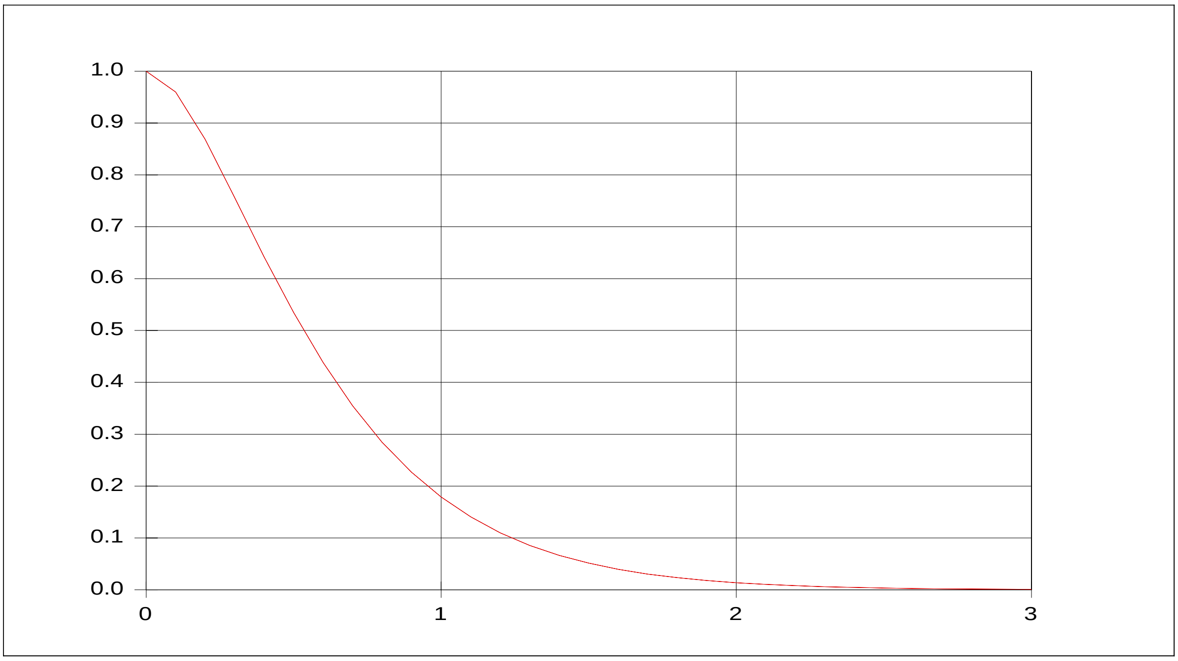
Have a look now at the two graphs, one superimposed on the other:



If you look closely you will see that the crossing of the y = 0 line is just a little later in the case of the damped oscillation (the red line) than is the case with the undamped oscillation. This shows that the frequency of the damped oscillation is slightly lower than that of the undamped oscillation. It is harder to tell where the maxima and minima are but I assure you that they are delayed as well.

Critical Damping

The damping can be increased from k = 0.5 but it must not exceed PI (unless we set up our spreadsheet to handle complex functions). The graph below shows what happens when k = PI. You will see that the amplitude never crosses the t axis.



Let's consider what this means for my tummy.

Initially I feel the loss of vertical 'g' forces as my tummy experiences free fall, however, if the damping is set so that k = PI then I feel only a little more than one 'g' vertically and my suspension never 'bottoms out'.

This value, k = r, is called "Critical Damping". There is nothing particularly 'critical' about it except that the 'real' number formula I have used breaks down if I make k any larger. In a practical physical system it is possible to increase the damping still further so that k > r. The graph still looks a bit like this last graph, the line does not cross the t axis, but it approaches the t axis more slowly taking longer than the 3 units (approximately - because y never truly reaches zero) shown in the graph.

Practical Systems

It will not have escaped your notice that for values of k lower than r such as k = 0.5 the curve crosses the t axis in a shorter time than it does for critical damping (but longer than for the undamped oscillation). However, there is some 'overshoot' (y < 0) when k < PI; the the positive 'g' experienced is higher than the maximum 'g' with critical damping.

Nevertheless, it is usual to design a vehicle so that the damping is slightly less than critical so that, when you drop off a road hump and hit the ground, your suspension will compress slightly and you will be subjected to slightly higher 'g'. It is a compromise designed to get your car ready for the next hump or bump in the road sooner than it otherwise would be.

One of the things you see people do when buying a secondhand car is to lean on the wing, release it and see how the suspension reacts. It should 'damp out' within one oscillation.

Finally

Some of you may have noticed that, if the car is full of people and holiday luggage, then there is less of a jolt but the car rides more deeply and oscillates more slowly (and for a longer period of time) when it encounters dips in the road. A suspension characteristic such as this is said to be 'soft'.

The design of the suspension must take account of the changes in resonant frequency which can occur when the load is changed. Heavier loads reduce the resonant frequency (longer time period) and lighter loads increase it. Generally, under all conditions, except the (overloaded) heaviest ones, the oscillation will damp out within one cycle. If it doesn't then you will find your passengers suffering from motion sickness and complaining that the ride is "too soft" meaning too many oscillations before it settles down.

At one time 'active ride suspension' was introduced into some Formula 1 racing cars to very good effect. I don't know whether it has become a feature of production cars yet but I think I seen some advertisements which say that.