

Gerald's Column by Gerald Fitton

During the last few months I hope that I have got you thinking about the meaning of probability and risk. Last month I concentrated on the technique of Expected Values. This month I want to discuss (with appropriate spreadsheet support) criteria which can be used when the outcomes are certain but the probabilities (used to calculate the Expected Values) are unknown or unreliable.

When to Use Expected Values

Using the technique of Expected Values (with variations such as the Value of Perfect and Imperfect Information) is useful if and only if the probability values are reliable (you have a high degree 'faith' in them) and if you base your decisions (the action you decide to take) on calculations of a similar nature and magnitude regularly enough for the 'good luck' and 'bad luck' to average out. I can not stress this enough. Techniques for decision making based on estimates of probability are valid only if similar risks are taken frequently enough for the long term result (of many similar decisions) to average out. Insurance companies do take such risks; they average out their risk amongst many customers.

Using the EV technique is totally inappropriate if the decision you make is either unique or rare. Never ever use the EV technique to decide whether you should take a life changing risk. Never ever use EV technique to justify a decision made about a rare event.

So what (calculated) criteria can you use as an aid to making your decision?

To Bet or not to Bet

I would like you consider whether or not to place a bet. This is a 'hypothetical' so the terms and conditions may seem a little strange—but please bear with me.

The odds offered are 300 to 1. You do not know whether these odds are 'fair' or not. By this I mean that you have no idea how likely you are to win or lose.

If you decide to bet then you have to bet £200. You can not bet more nor can you bet less.

There are three criteria for making your decision; these are usually called MaxiMax, MaxiMin and MiniMax. This last criterion is sometimes represented differently from a straight MiniMax criterion; in this case it is called the criterion of Least Regret. I shall deal with the MiniMax technique using the Least Regret formulation.

Least Regret

The screenshot below shows part of the Fireworkz sheet [Betting01].

It contains a Betting Outcome Matrix and a Regret Matrix. The first (the Outcome Matrix) lists the results of the various outcomes and the second matrix (the Regret Matrix) quantifies your loss if you make the wrong decision.

From the screenshot you'll see that the value in cell c11 is calculated by subtracting the value in c5 from the maximum value in that column. Because of the negative 200 in c5 it does make the interpretation of the result (in c11) harder to interpret than the value in d12.

The screenshot shows a spreadsheet with the following data:

	a	b	c	d	e
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					

The spreadsheet contains two matrices:

Betting Outcome Matrix

	Betting	Outcome	Matrix
<u>Action Taken</u>		<u>Lose</u>	<u>Win</u>
Bet		-200	60 000
Don't Bet		0	0

Regret Matrix

	Regret	Result	of Bet
<u>Action Taken</u>		<u>Lose</u>	<u>Win</u>
Bet		200	0
Don't Bet		0	60 000

The formula bar shows: $\text{max}(c5:c6) - c5$

The interpretation of the Regret Matrix is that if you bet and lose then you regret the loss of your stake money, £200, whereas if you don't bet and could have won then you regret the lost opportunity to win £60 000!

The MiniMax criterion aims to minimise the regret which you feel when you know the result. Those 'punters' who believe in the minimum regret criterion would place the bet in order not to regret the (rare?) occasions on which they would win.

The question I would ask myself is whether I could afford to lose £200. This £200 is a genuine loss of something I have. The £60000 is 'pie in the sky' and I would regard it as a windfall if I won rather than a loss if I didn't win. So I wouldn't place the bet.

Others for whom £200 is trivial and for whom £60000 is substantial enough might regret the loss of £60000 more than I would.

All the computer can do is the sums. You have to decide what is important to you.

MaxiMax and MaxiMin

In the screenshot below I have extended the spreadsheet to show the MaxiMax and MaxiMin calculations. I have left the Regret Matrix calculations in place.

The formula in g5 is simply $\max(c5d5)$. You'll see that the £60000 is returned. Similarly the formula in g6 is $\max(c6d6)$. The formula in g10 is $\max(g5g6)$ and the formula in g11 is $\min(g5g6)$ as shown in the screenshot.

Betting Outcome Matrix		Regret Matrix		Decision Matrix		
Action Taken	Outcome	Result of Bet	of Bet	Criterion	Value	Choice
Bet	-200	200	0	MaxMax	60 000	Bet
Don't Bet	0	0	60 000	MaxMin	0	Don't Bet

Sometimes the MaxiMax criterion is called the Optimist's Criterion. An Optimist will Bet because he hopes to win.

The MaxiMin criterion is sometimes called the Pessimist's Criterion. A Pessimist will not Bet because he expects to lose. By not betting he will 'save' £200 when the result is a loss.

Gerald's Second Rule of Gambling

You will remember that my First Rule was: "Never bet on certainties." I arrived at this rule by considering Conditional Probabilities; I asserted that there is no such thing as Absolute Probability.

My Second Rule is: "Never bet what you can't afford to lose".

In any situation I recommend that you consider the MaxiMin (the Pessimist's) Criterion before considering the others I have mentioned.

If you can afford to bet the £200 and lose it then you will have 'passed' this first criterion.

Be careful, don't convince yourself that it is a 'risk worth taking' by looking at the possible rewards of winning (that you can do later), consider only what your situation will be if you lose the £200. If you can't afford to lose the £200 or if the £200 could be better spent elsewhere or if spending it on this bet would cause you to miss another (better) opportunity elsewhere then your choice must be not to make the bet.

The MaxiMax Criterion

Only after passing the Pessimist's Test should you consider this criterion.

Now you can look at what happens if you win. Often you will have more than one choice for the expenditure of your £200. Regardless of the chances of actually winning which, in my hypothetical, you don't know, do you really want to have a go at this bet which returns £60000 or should you try one which returns a lot more or a lot less?

Umbrellas or Parasols

Now just a brief look at the Umbrellas and Parasols. No longer do you go to the market every week. You have one and only one chance to make a 'quick buck'. You can take Umbrellas or Parasols. Which do you decide to take?

The screenshot shows an Excel spreadsheet with the following data:

Outcome	Sales	Matrix	
	Type of	Weather	Max
Item taken	Dry	Wet	
Umbrellas	750	6 000	6 000
Parasols	4 000	1 000	4 000

Least	Regret	Matrix	Decision	Matrix	
	Type of	Weather	Criterion	Value	Choice
Item taken	Dry	Wet	MaxMax	6 000	Umbrellas
Umbrellas	3 250	0	MaxMin	4 000	Parasols
Parasols	0	5 000			

First of all, it would appear that you are in a win-win situation. So, whatever you do, you are not betting what you can't afford to lose. Hence forget about the MaxiMin criterion.

We have no ideas about the probabilities so we'll forget about Expected Values.

The 'top prize' (from the MaxiMax criterion) is 6000 when you take Umbrellas.

That sounds pretty good but let's have a look at the Regret Matrix too. So far as Regret goes (if you make the wrong choice) what will happen is this. If you choose Parasols and it is Wet then the Regret is worth 5000 units whereas your Regret is less (at 3250 units) if you choose Umbrellas and it turns out Dry. On the basis of the Regret Matrix the 'correct' choice is also Umbrellas.

So it looks as though Umbrellas is the the 'best' choice.

It just so happens that Umbrellas is the 'optimum' EV choice as well—but that's just the way the numbers happen to turn out. It could easily have been different.

Should you insure?

In the first hypothetical (above) I have referred to placing a bet. Not all bets are taken with a bookmaker. Some bets are taken by insuring—or not insuring.

Should you insure your house against a total disaster?

If you do insure your house you are placing a bet. Your insurance premium is your stake. You 'win' your bet if your house is destroyed and you make a successful claim on your insurance. You lose your stake money if your house remains totally incident free and, as a consequence, you can not claim your 'prize money'.

Now here is an important point. If you do not insure your house you are still placing a bet. This time the stake is much higher. Your 'stake' is the value of your house.

Let's forget totally about Expected Values because it is not relevant. Unless you are very rich then your situation is that you can afford to lose your stake money if you do insure but you will be destitute if you do not insure and your bet goes wrong.

The most relevant criterion here is the MaxiMin (Pessimist's) Criterion. You can not afford to lose your house but you can afford to lose your insurance premium in a failed bet.

Now let's have a think about Expected Values. The insurance company will do lots of Expected Value sums and, using that technique, will decide on an appropriate premium. This EV technique is appropriate for them because they have enough customers to be able to average out their good and bad luck.

Let's continue. There is no doubt that, in most cases, the bet of your insurance premium almost certainly will not be a 'fair bet' because the insurance company will charge a little more than the probabilities justify. If you were knowledgeable enough to know the probabilities then you'd find that, on the basis of Expected Value calculations alone you should not insure!

However, your EV calculation is irrelevant to your situation. Unless you are very rich and own a lot of properties ('you' might be the local Council or Housing Association) then, the MaxiMin Criterion is paramount and you must insure.

As an aside let me ask a question. If you did find an insurance company who did offer a substantially lower premium than the probabilities indicate is 'fair' then would you bet your stake (your insurance premium) with them? Personally I would not. I would like to be sure that the insurance company is making enough profit to still be in existence when the time comes for me to make a claim! If they don't charge a premium which is higher than the probabilities (EV calculations) indicate then they might not last long enough to honour my valid claim.

Summary

I am not a great believer in Expected Values for making personal decisions when the circumstances are unique or rare.

The Pessimist's Criterion (MiniMax) is the most important of those which do not require known and reliable probabilities.

Remember Gerald's Second Rule of Gambling—don't bet what you can't afford to lose!