Gerald's Column
by Gerald Fitton

Thanks for your letters and emails. Thanks also for your thoughts about what I have called Problems of the Second Kind; these are problems which, for their solution (within a reasonable period of time) require more subtle algorithms than those which I have called Brute Force Methods.

In considering how I might follow up this theme, and after some thought, I decided to take you on a rather longer journey in search of this 'El Dorado' because, like Robert Louis Stevenson (1850–1894), I believe that travelling hopefully may be of wider appeal and ultimate value ("and the true success is to labour") than would be your arrival at that destination without having experienced the arduous task of studying the scenery on the route.

Invention or Discovery

Mathematicians were responsible for the early development of the computer so it is not surprising that the way in which they work owes a lot to mathematical thinking. I shall show you the way in which a bias towards mathematical thought has strongly influenced the way in which the current range of computers work. I will do this so that you can free yourself from the prejudice of mathematics and in the sure and certain hope that, when you are free, you will appreciate more readily how a new sort of computer might be developed.

The distinction between Invention and Discovery is blurred more often than is realised.

My guess is that all but a handful of you believe that computers were invented. Computers were not in existence just waiting to be discovered. I would agree with you. On the other hand the fact that water is a compound of hydrogen and oxygen was always true but we didn't know it until electrolysis was invented by Sir Humphrey Davy (1778–1829). The chemical nature of water was a discovery and not an invention.

There are other entities for which the distinction is not so clear cut. Music is one such example. Is music a natural phenomenon which lay dormant for eons waiting to be discovered or does music exist only in the mind of man? Is music man's invention?

Mathematics is another entity for which the provenance is doubtful. There is a belief widely held by non mathematicians that mathematics would not exist if there were no mathematicians to appreciate it. This is a view which is not held by many present and past prominent mathematicians. In particular many mathematical physicists of great repute often follow Galileo Galilei (1796–1873) in their belief that "Nature's great book is written in mathematical language". Sir James Hopwood Jeans (1877–1946) is often quoted as the originator of this creed but, in his Rede lecture delivered before the University of Cambridge in November 1930, Sir James gives credit to Galileo before saying ". . no one except a mathematician need ever hope fully to understand . . . the fundamental nature of the universe". I would have added one word to Sir James' statement and say that the fundamental nature of the ***physical*** universe is mathematical. Many prominent mathematicians and physicists, when in reflective mood, assert that "The creator of the universe was a pure mathematician"! So my enigmatic and rhetorical question to you is "Did we invent mathematics or only discover this aspect of our creator?"

Everything which has been invented contains some part of its creator. Mathematicians developed the computer and, as I shall demonstrate, the computer of today shows its mathematical provenance.

Numbers

The basis for many areas of Mathematics is numbers so that is where we shall start our quest. The current type of computer does everything with and by numbers so I shall use that as my tenuous justification for spending a few happy hours telling you about Numbers.

Mathematicians know of many types of number but I shall concentrate on just five, the ones shown in the table below. I shall spend a little time with each type of number, describing what they are, how they are related to other sorts of numbers and, because this is a computer magazine, how computers handle these numbers.

The table

Sometimes when I include something such as the table below in an article I am asked for details of its creation.

$\mathbb{N}$	=	{0, 1, 2, 3, 4, 5, 6, . . . }		=	Natural (counting) numbers
$\mathbb{Z}$	=	$\mathbb{N}$	+ {0, −1, −2, −3, . . . }	=	Integers (whole numbers)
$\mathbb{Q}$	=	$\mathbb{Z}$	+ {¼, ½, ¾, . . . }	=	Rational numbers (fractions)
$\mathbb{R}$	=	$\mathbb{Q}$	+ {$\pm\sqrt{2}$, π, e, . . . }	=	Real numbers (smooth continuum)
$\mathbb{C}$	=	$\mathbb{R}$	+ {$\sqrt{-1}$, (1 + $\sqrt{-1}$), . . . }	=	Complex (and imaginary) numbers

I created this table in DrawPlus. Afterwards I had another go using Equasor and then TechWriter. I was successful with all three packages. Indeed, it is my view that the ease or difficulty of construction of this table in any of these packages would depend only on the extent to which you are familiar with the package. Equasor and TechWriter would come into their own if the mathematical statements were more complex.

I used DrawPlus rather than Draw primarily because I find that the text editing features of DrawPlus are far superior to those of Draw. Furthermore, in DrawPlus there is a useful feature which allowed me to align all the equal (=) signs and arrange the statements so that they were vertically equidistant from each other.

The font I used for such things as “1, 2, 3” etc is Trinity. The symbol for pi, “π”, and “$\sqrt{\ }$ ”, the square root symbol, are in MathPhys font. Finally, the standard mathematical symbols for the different types of numbers namely $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, and $\mathbb{C}$, are in a font called MathScript. This is the font used for these sets of numbers in Mathematical text books. When writing in longhand most but not all of the hollow areas are retained, for example the left vertical of the $\mathbb{R}$ is retained but not those extra lines on the right hand side of the $\mathbb{R}$.

Although the equal sign is used in the table, the curly brackets, {}, show that this is not an ordinary equation in which ℕ is some variable or constant number the value of which might be found by solving an equation. In words the first mathematical statement should be read as "The symbol ℕ represents the set of all Natural (counting) numbers such as 1, 2, 3, 4, 5, 6, etc". The second statement can be read as "The symbol ℤ represents the set of all Integers; this includes all the numbers in the set ℕ and, in addition, all the negative integers as well."

The Natural Numbers

I have no doubt that the first number to be discovered was the number two. There is no need for a number when the entity is unique. I suspect that the number one followed close on its heels and then all the larger numbers that were required. The Ancient Babylonians used the Natural numbers to count but their system differed from ours in that they didn't carry one (into the tens column) until they got to sixty. It is for this reason that we have sixty seconds in a minute and sixty minutes in an hour.

The number zero was a late invention. The Romans used Natural numbers in their system but what is their symbol for zero? They didn't have one!

I would like you to imagine yourself as an accountant in Roman times. One of the exercises which I have set students is to subtract the year 1982 from the current year using Roman numerals. Imagine keeping your accounts in Roman numerals!

Place notation is the use of units, tens, hundreds, etc, columns; it is impossible to construct numbers in this notation without a symbol for zero. Place notation is used by computers and is an indispensable part of floating point arithmetic. Although we are not sure, it seems most likely that zero originated in Arabia about a thousand or so years ago before moving to India and then via the Venetian traders to us. As recent as the eleventh century a law was passed requiring all accounts for tax purposes to be held in Roman and not Arabic numbers. This law was never repealed and the Venetian traders of the twelfth century (like many traders today) kept two sets of books, one for their own benefit in Arabic numerals, the other for tax purposes in Roman numerals! It took over a hundred years before those people doing sums for a living in the Western world were converted to place notation.

I remember as a child noticing that the copyright year of films was always in Roman numerals – I was told that it was a legal requirement!

Whether your computer is doing sums in binary or in binary coded decimal, integer or floating point, it uses place notation and place notation is impossible without a symbol for zero.

Is Zero a Natural number?

For many centuries there has been argument amongst mathematicians about zero. Indeed, those who were brought up on Roman numerals questioned the very existence of zero. It was only this century that Bertrand Arthur William Russell (1872–1970) finally proved that it was more logical to include zero amongst the Natural numbers than leave it outside that set. His argument put in a slightly oversimplified way is that you can count that zero people are in a room if you are able to understand the meaning of the question "How many people are there in this room?” when nobody is there. If you had no idea what is meant by ‘whatnots’ then you would be hard put to recognise whether there were any there or not! If you weren’t sure where the boundary of the ‘designated area’ was then you would have an equal problem.

I guess you would all accept that you can’t count how many people there are in a room if you start with three and five walk out. Negative Integers are not Natural numbers.

Counting

My son tells me that I’m not a ‘Computer person’. With some irony he defines a ‘Computer person’ as one who starts counting the number of people in a room as “zero, one, two, etc,”. You might not do that but if you have programmed in machine code or assembly language then you will recognise what he is getting at. I shall explain.

As a mathematician the difference between the method of counting my son describes and the more usual method is reflected in the difference that exists between Ordinal and Cardinal numbers. The traditional method of counting is to hand out labels to each person in some chosen order starting with a label which has “one” written on it. When you have finished handing out the labels you could get the recipients to ‘stand in line’ (as my American colleagues would say). The numbers written on their labels are Ordinal numbers; the people would stand in the order of the numbers on their labels. Now you look at the label held by the last person in the queue and the number written thereon is the number of people in the ‘set’. That number is called the ‘Cardinality of the set’ – the number you deduce from looking at the last Ordinal number is a Cardinal number.

You can do sums with Cardinal numbers (three people plus two people equals five people); you can not do sums with Ordinal numbers.

When you program in assembly language then, at some time or other, you will have to reserve a block of memory in which to store bytes of data. The method (with all computer systems I know of) is to refer to a particular memory location by the address of the first available ‘slot’ and an ‘offset’ from that slot. Even BBC Basic uses this convention. Those of you not accustomed to assembly language programming may be surprised to learn that the first available slot has an ‘offset’ of zero! The third available slot has an ‘offset’ of two!

In a previous existence I used to teach assembly language programming. Getting the students to understand that the memory block had not ‘lost’ nor ‘gained’ one slot was a major hurdle. For example if you reserve a block 256 bytes long then the block is full when you get to the slot at the ‘offset’ labelled 255.

Even if you are not into assembly language programming you will have heard of ASCII and know that there are 256 codes available and that the ‘last’ one is ASCII Code 255 (and not number 256). That is because there is an ASCII code zero, &00 in hex, called NUL which usually does nothing.

So there is some truth in my son’s definition of a ‘Computer person’ as one who starts counting the number of items in a set by labelling the first item as the zeroth item.

More

I am tempted to write more about Natural, counting numbers this month but I shall desist and leave until next month such spreadsheet functions as count(), dcount() and dcounta() which return a Natural number as their value. Instead I shall return briefly to my theme.

Mathematics

Although for many years I earned my living as an engineer by nature I was always a scientist so they called me a ‘Technologist’. However, my early qualifications were in neither; my primary academic qualification was in Mathematics.

It is said by those who study the History of Mathematics that mathematicians do their best work before they are 30 years old. My own interpretation of that history is not at odds with this contention but I would add that many prominent Mathematicians turned to Religion and Philosophy in their more mature years. Bertrand Arthur William Russell (1872–1970) was the mathematician who proved that the basics of arithmetic followed from logic and solidly established the number zero as a counting number (see above), he turned to Philosophy whereas Isaac Newton (1642–1727) turned to Religion. René Descartes (1596–1650) of Cartesian Coordinate fame became a Philosopher.

The Future of Computers

Whilst my contributions to Mathematics is minimal I have found that as the years have rolled by my Mathematics and Science are tempered with Philosophy which I’m sure shows through my writings on occasions. Hence, in my more quiet moments, I consider the extent to which computers are what they are today just because Mathematicians were responsible for their early development. Then I get to wondering how long it will be before less logical machines will be developed. In my time I have read many books about cybernetics and many of them discuss artificial intelligence; their authors consider how we learn by experience rather than by a pre programmed response to a stimulus. One book which you might find readable is “Design for a Brain” by W Ross Ashby who is an authority on Cybernetics. It is eminently readable for many reasons only one of which is that the mathematical content is almost non existent.

Certainly if those of like mind to W Ross Ashby had invented and developed the computer then it would owe little to mathematics and much more to that branch of systems engineering which relies on negative feedback – learning from experience.

At this point I shall not develop further the concept of illogical computers and their uses. Instead I shall await with some glee the mail which I have no doubt that I shall receive on this and other related topics. I shall be pleased to read those thoughts whether they are about the nature of Computers, Mathematics or even the nature of the physical universe in which we, at least for a while, work out our existence.