Gerald's Column
by Gerald Fitton

I am still getting many requests for more detail instructions for the creation of graphs from data held in spreadsheet packages. In this month's article I shall try to explain how to create the type which you find most difficult, namely a multiple Line Graph from columns of values of X and Y. I have only enough space to show how to do this using one spreadsheet package so I shall use my favourite package, PipeDream. I shall demonstrate how to create similar graphs using other packages on another occasion.

Another pair of topics which has led to much correspondence is the mathematical concept of Infinity and its time related counterpart, Eternity. It pleases me that so many of you are so interested – it surprises me that I have had no negative comment (perhaps it has gone directly to Paul). Anyway, this month the philosophical bit comes at the end starting at the section headed "Acknowledgements".

The Infinite in a Point

If I were to ask you to specify your favourite function it is very likely that you would give me a formula for the function. With such a formula – for example $y = \sin(x)$ – you could calculate the value of y for any value of x I might care to choose.

Whilst that is a perfectly good way of specifying your function there is another way. Instead of having a formula which specifies directly (through the function) the value of y for any given value of x we can concentrate on just one value of x and find out lots of things about that one point. For my example I shall concentrate on the single point $x = 0$.

Have a look at the fourth row of the table below. It contains everything there is to know about my chosen function (have a look at the title and guess the function) at the point $x = 0$ – and it contains nothing about any other point. Although I'll explain the interpretation of the numbers in that row later, for now I must tell you that I'm cheating a bit because, to get an accurate result for extrapolation to all possible values of x, I need a fourth row which contains an infinite number of columns! Of course I can't have an infinite number of columns on my computer (and neither can you on yours) so we'll have to make do with just a few (I've used fifteen) and accept that the answers will be approximate!

PipeDream: SCSI::HD4.$.PipeLine1.Archive.9609.Sine.PipeDream.Sin_x *

	A	D	E	F	G	H	I	J	K
1									
2	n	0	1	2	3	4	5	6	7
3	n!	1	1	2	6	24	120	720	5040
4	f(0)	0	1	0	-1	0	1	0	-1

So, here's my claim: using only information about the function at $x = 0$ (the information in the fourth row), plus the assumption that the function is 'regular' (smooth), we can use a spreadsheet to find the value of this function for any value of x between minus infinity and

plus infinity.

The series generated from the data in row 4 is shown in the column below. I'll explain how the sums are done later. Only part of the table is shown in the screenshot. The table extends to row 58 so that values of y are calculated from x = 0.00 to x = 7.50.
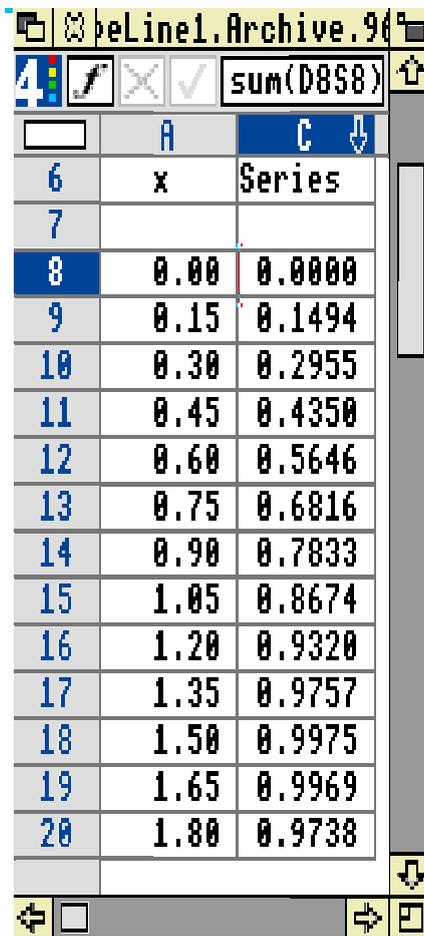
	A	C
		sum(D8S8)
6	x	Series
7		
8	0.00	0.0000
9	0.15	0.1494
10	0.30	0.2955
11	0.45	0.4350
12	0.60	0.5646
13	0.75	0.6816
14	0.90	0.7833
15	1.05	0.8674
16	1.20	0.9320
17	1.35	0.9757
18	1.50	0.9975
19	1.65	0.9969
20	1.80	0.9738

Title bar: eLine1.Archive.9...

It is said that a picture paints a thousand words (and I notice that Paul is asking for more of them in Archive) so let's delay some of the explanation and jump to the graph of this function for values of x from 0.00 to 7.50.
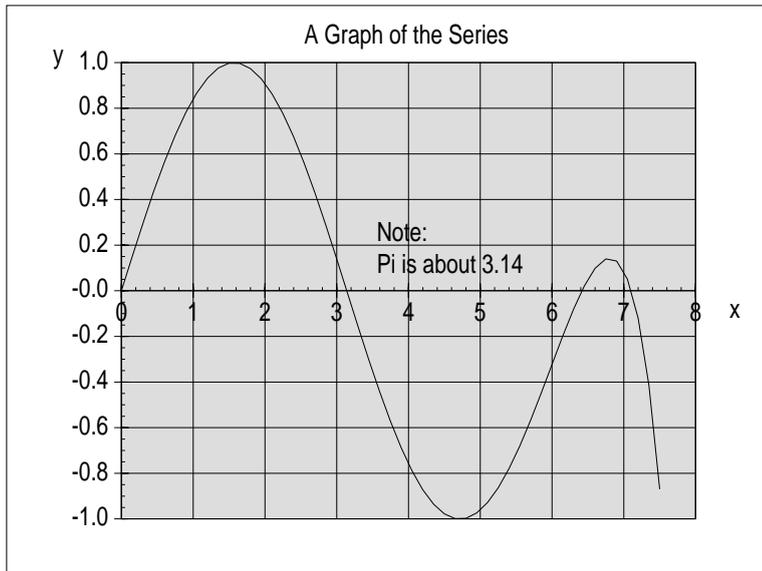
The graph, produced in PipeDream but saved as a DrawFile, is shown above. I would like you to note that the function appears to have a maximum value of 1.00, a minimum value of −1.00 and crosses the axis where x = pi. I will explain how to produce this graph in some detail later.

The Swing

You may doubt the validity of the process of using lots (an infinite amount!) of information about a point on a curve to calculate (with a simple spreadsheet) the value of the function at any other point. Let me try to convince you.

As an example, let me help you to construct a mathematical model of the action of a swing. In this model x is time and y is the position of the swing. When the swing is in the middle y = 0; when the swing is in its most forward position y = +1; and when it is as far back as it will go then y = −1. We start the clock at x = 0 with the swing in the y = 0 position.

The conventional way of constructing the mathematical model is to specify the function which allows us to calculate the position of the swing at any time. A good approximation for this function is y = sin(x). A better one would be a damped oscillation but let us keep it simple; let us choose to have our swing oscillating forever between the same extreme positions, y = −1 and y = +1. It will be travelling at it highest speed, +1 velocity units, when y = 0. It will be subject to maximum acceleration when y = −1 and when y = +1 but zero acceleration at y = 0.

I would like you to consider the following alternative method of creating the mathematical model using only that information which can be gleaned at the time x = 0. Where is the swing? Answer, y = 0; it is in the middle of its swing. Now please refer back to row 4 of the original spreadsheet; in column D (under n = 0) you will find the value 0. The meaning of this 0 is that y = 0 when x = 0. Next, how fast is it going as it passes through the point y = 0? Well, we'll suppose it to be +1 in our arbitrary velocity units. Refer to column E of the spreadsheet where you'll find the value 1. This 1 means that the velocity at time x = 0 is 1 unit. Next let us ask whether the swing is speeding up or slowing down? In fact there is no horizontal force on the swing at the the lowest point so it is neither speeding up nor is

it slowing down. You'll find a 0 in column F. However, although the acceleration at the lowest point is zero, it is in the process of change. Yes! I know this is a hard concept to grasp but, as the swing goes through its lowest point, the force on it changes from a 'speeding up' force (pulling the swing towards the middle) to a 'slowing down' force (being exactly zero at the lowest point). Please accept from me that this rate of change of acceleration is −1 in our rate of change of acceleration units; hence the −1 in slot G4. We could continue this discussion but it does get harder to conceive as we get to higher order 'rates of change'.

For those of you into differential calculus you will recognise that the values in row 4 from column E onwards, are the first, second, third, etc derivatives of the function y = sin(x) evaluated at x = 0. You could continue this process of evaluating higher order derivatives for ever but you don't need to because the pattern of 0, −1, 0, +1 repeats indefinitely – so you don't have to keep working them out forever.

The Series Approximation

Whether you are into calculus or not I hope that you'll recognise that, if you know such things as position (y = 0), speed, acceleration, rate of change of acceleration, etc, at just the one point, x = 0, you do know quite a lot about what will happen to the swing in the future. Indeed, the more you know about the situation at x = 0 (the more columns of row 4 you can fill in) the further into the future you can predict the motion of the swing. Our chosen mathematical model of the swing is that it never stops swinging; the values in row 4 keep repeating the 0, −1, 0, +1 pattern infinitely.

If I knew everything (an infinitely long row 4) about a function at one point, x = 0, then (with the additional assumption of smoothness) I would be able to predict future values of y for all values of x up to infinity! Of course, if the smoothness condition is violated by somebody interfering with the swing, pushing it harder or stopping it, or if the rope breaks (!), then smooth extrapolation from x = 0 breaks down.

What I want to show you now is how you can use a spreadsheet to feed the values of the derivatives in row 4 into a calculation which finds the values in column C from which the graph is plotted. However, before I do that, first let me show you a graph of y = sin(x) and a graph of the function which is produced by approximating the infinitely long row 4 with just fifteen terms.

The graph below shows the 'original' function and its fifteen term series approximation. You will see that the approximation is a fairly good one up to about x = 5 or 6. It would extend further if more terms were included in row 4.

A Series Expansion for Sin(x)

y = Sin(x)

x in radians

Note:
pi radians = 180 degrees

Series Approximation
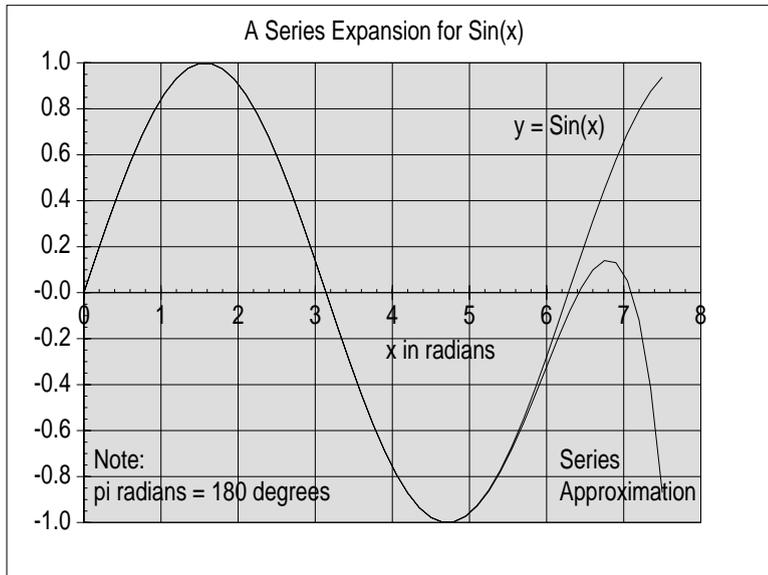
From a Point to a Function

The spreadsheet shown in the screenshot below is the one I've used to calculate column C from the values in row 4. As I will show you later, you can change the values in row 4 so that row 4 represents a different function. If you do this then all the dependent values below row 7 will change and, if you have a live graph which plots column C, then the graph will change automatically to display the new function.

PipeDream: scsi::HD4.$.PipeLine1.Archive.9609.Sine.PipeDream.Sin_x

$A8^E$2/E$3*E$4

	A	B	C	D	E	F	G	H	I	J	K
1											
2	n			0	1	2	3	4	5	6	7
3	n!			1	1	2	6	24	120	720	5040
4	f(0)			0	1	0	-1	0	1	0	-1
5											
6	x	sin(x)	Series	Term 0	Term 1	Term 2	Term 3	Term 4	Term 5	Term 6	Term 7
7											
8	0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	0.15	0.1494	0.1494	0.0000	0.1500	0.0000	-0.0006	0.0000	0.0000	0.0000	-0.0000
10	0.30	0.2955	0.2955	0.0000	0.3000	0.0000	-0.0045	0.0000	0.0000	0.0000	-0.0000
11	0.45	0.4350	0.4350	0.0000	0.4500	0.0000	-0.0152	0.0000	0.0002	0.0000	-0.0000
12	0.60	0.5646	0.5646	0.0000	0.6000	0.0000	-0.0360	0.0000	0.0006	0.0000	-0.0000

The formula in slot E2 is D2+1. The formula in slot E3 is D3*E2. Both these formulae are copied ('replicated' or 'filled' depending on the spreadsheet you're using) to the right as far as you want to go. I have extended my spreadsheet to column S. You will see from the screenshot that the formula in E8 is $A8^E$2/E$3*E$4. This formula can be copied into the whole block D8S58 with the exception of D8 into which it is better to enter the formula 1/D$3*D$4 because of the difficulty some spreadsheets have with 0^0. The formula in C8 is sum(D8S8) and this can be copied down the whole of column C.

Using the notation that $f^n(0)$ is the nth derivative of the function y = f(x) evaluated at x = 0 and that n! is factorial n (eg 7! = 7*6*5*4*3*2*1), the mathematical formula for converting from derivatives (row 4) to values of the function (column C) is:

$$y = f(0) + x \times f^1(0)/1! + x^2 \times f^2(0)/2! + x^3 \times f^3(0)/3! + \ldots + x^n \times f^n(0)/n! + \ldots$$

Drawing the Graph in PipeDream

The graph headed "A Series Expansion for Sin(x)" has been produced using PipeDream.

To produce a graph in PipeDream you must first mark a block. Mark the block A8A58. Execute the command <Ctrl CHN> (CHart New). You will find yourself with a bar chart – not exactly what you want. Never mind; we can easily change it.
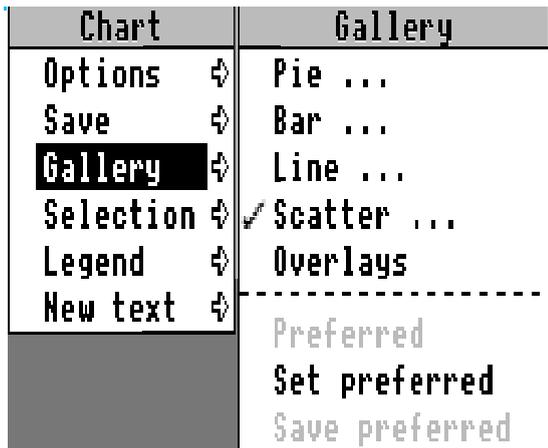
Place the pointer over the bar chart and click the <menu> button. The Chart menu will pop up. Run the pointer through Gallery and then click on Scatter to bring up the gallery of Scatter graph types.
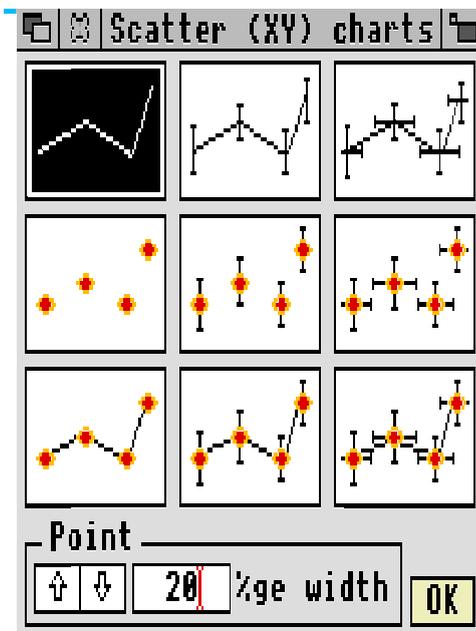
Select the top left type of Scatter (XY) chart (it is shown in inverse video in the screenshot) followed by the OK box. Not much will happen but don't worry about that.

In the spreadsheet mark the block B8B58. These are the y values of the function y = sin(x) corresponding to the values of x, A8A58, already entered into the chart. Be careful with the next bit. Execute the command <Ctrl CHA> (CHart Add), which, if you use the Chart pop up menu (from within the spreadsheet, not with the pointer in the Chart) is entitled "Add to chart". The rather blank looking chart will be converted into a line graph of the function y = sin(x).

Adding the second line

We want to add the second line, the series approximation, column C, to the chart.

First highlight (mark) the X values, A8A58, and execute <Ctrl CHA>, "Add to chart". Mark the Y values, C8C58, and execute "Add to chart" again. The second line will be drawn on the same chart.

Although in this case I have used the same range of X values for both lines, in PipeDream it is possible to use different X ranges on the same chart. What you have to remember on an XY chart (misleadingly called a Scatter chart in PipeDream) is that as you "Add to chart" the marked ranges are successively X, Y, X,Y, X,Y etc.

Adding Text

Adding text in PipeDream is a real pleasure compared with some other spreadsheets.

Click the <menu> button over the chart to call up the Chart menu. Run the pointer through New text and a Text window will appear. You can type multi line text into this window. To start a new line as part of the same text object you must <select> the blue left pointing arrow just to the right of the cross and the tick. In the screenshot the words "Series Approximation" (on two lines) has been entered into the Text window. When you tap <Return> or click on the tick, the words in the window will appear on the chart. You can move the text object about on the chart until it is positioned exactly where you want it using a select and drag technique.
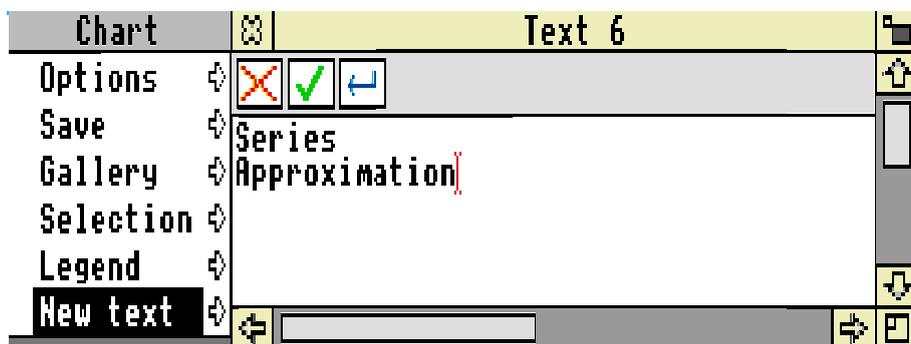
If you find that you have made a mistake then you can select the text object, click <menu> and then run the pointer through Selection, Text and then Edit. After editing click <Return> to return the modified text object to the chart. You will notice that you have an option to Delete the text object if you wish.

If you select a text object and then Selection on the Chart menu, you can change the Text style (font and font size), Text colour or Text background.
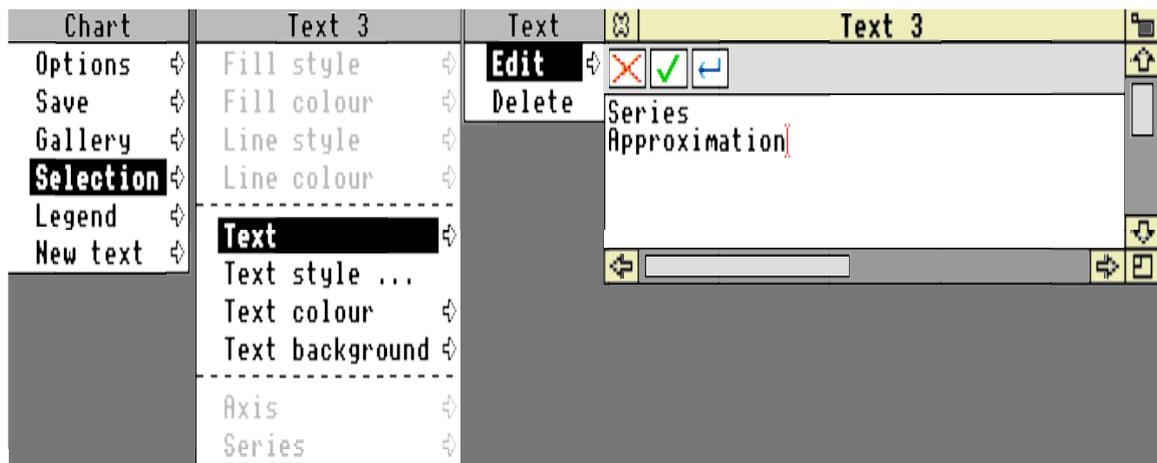
Saving

Don't forget to Save the chart separately from the spreadsheet. You will find the files on the Archive monthly disc. I have saved the spreadsheet as [Sin_x] and the chart as [Sin_x_C]. This name for the chart uses the convention recommended by Colton Software, namely adding _C to the spreadsheet file name.

On the Archive monthly disc I have also saved the chart as a draw file under the filename [Sin_x_G]; G stands for Graphic.

Other Functions

A quick change to the spreadsheet allows other functions to be calculated. Have a look at the spreadsheet in the screenshot below.

The only difference (apart from the file name) is that I have changed all the values in row 4 to +1. This set of derivatives generates the function $y = \exp(x)$. The chart used with the $y = \sin(x)$ was still alive and changed appropriately to that which you see in the drawfile.

A Series Approximation to exp(x)
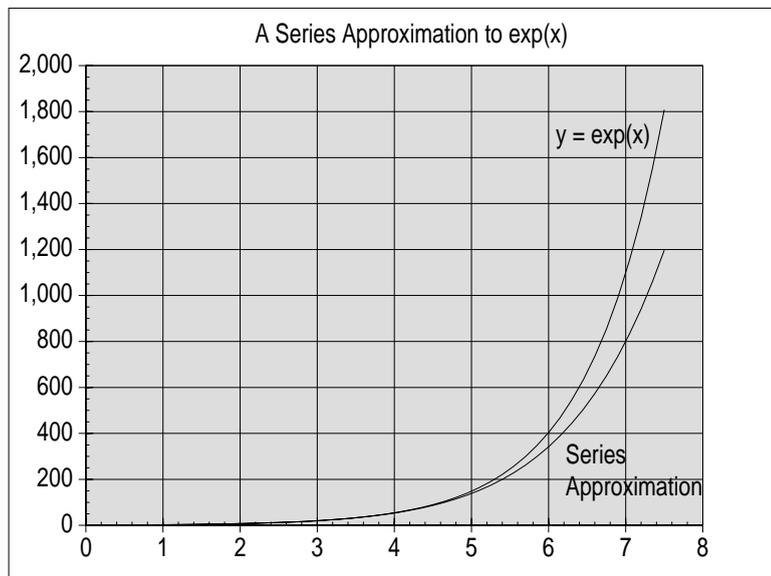
I have to confess that, when I first generated this chart it seemed as if there was only one line on it; the approximation was that good. So what I did was to change the formulae in column C of the spreadsheet to sum only eight (rather than fifteen) terms of the series. This makes the approximation a little worse and ensures that there are two separate lines displayed on the chart.

You might like to try other sets of derivatives such as the series 0, 1, 0, 1, 0, 1, etc (which generates a function you might be able to recognise). If you are good at recognising some of the standard functions then try 0, 1, 1! 2!, 3!, 4!, etc (ie use $(n-1)!$ in row 4) and try the series 0, +1, 0, −2!, 0, +4!, 0, −6!, etc; both of these generate standard functions available in any spreadsheet or on any calculator.

By the way, have you ever wondered how calculators (and spreadsheets) produce values for sin(x) or exp(x)? No! They don't hold a table of values of sin(x) or exp(x) – they are evaluated in the calculator using suitably truncated infinite series.

Acknowledgements

One of the earliest philosophies by which I tried to live my life was to pursue 'truth'. Nowadays it is fashionable to believe that 'truth' is a relative term so perhaps what I have to say about eternal truths is somewhat unfashionable. One of the features of mathematics which I found attractive even whilst at Junior School, is that, once proved, mathematical truths tend to last. In a world of changing values, mathematical truths remained true. I have often said that, if you want to be remembered for ever then you must discover a new mathematical theorem. At some time in the far future, when Einstein's Theory of Relativity (and his name) has been long forgotten, the navigator of a lonely space ship will still use Pythogoras' Theorem (about the square on the hypotenuse) and still call it by that ancient mathematician's name.

At college, I teach nothing mathematical which is less than 100 years old and most is 300 years old or (like complex numbers, negative numbers and Pythagoras' Theorem) much older. My first acknowledgement is to Sir Isaac Newton who, one long hot summer in 1666, whilst on an extended holiday from his 'Chair' at Cambridge university, invented

what we call the Infinitesimal Calculus and he called his 'Theory of Fluxions'. By the way, the university's summer holiday was extended that year because of The Plague. As another 'by-the-way', he invented the Integral Calculus before the Differential Calculus; nowadays we teach differentiation before integration!

Next I must acknowledge that the formula

$$y = f(0) + x \times f^1(0)/1! + x^2 \times f^2(0)/2! + x^3 \times f^3(0)/3! + \ldots + x^n \times f^n(0)/n! + \ldots$$

is due to a Scottish mathematician called Colin MacLaurin who was given a 'Chair' at Edinburgh (thanks to a recommendation by Newton a couple of years before he died) in 1725. The so called MacLaurin expansion (the above formula) is a simplified version of one developed by the Cambridge mathematician Brook Taylor and originally published in 1715 in his treatise "Methodus Incrementorum" (in those days books intended to contain Eternal Truths were always written in Latin) in which he sets out the method I have tried to explain to you.

Mathematical 'truths' last a long time – I believe them to be amongst the Eternal Truths.

Finally, I must acknowledge that the real inspiration for the philosophical content of this article comes not from a mathematician but from *Auguries of Innocence* by the poet, William Blake (1757 – 1827) who wrote Jerusalem and "Tiger tiger burning bright".

To see a World in a grain of sand
And a Heaven in a wild flower
Hold Infinity in the palm of your hand
And Eternity in an hour

Whilst I would have preferred Blake in his augury to have foreseen the nature of Infinity in a single Point and the nature of Eternity in an infinitesimal Moment I accept that his alternatives are more poetic. My philosophical point is that, by studying just one point (x = 0) of a mathematical function in great detail then, with the assumption of regularity (smoothness), one can find the value of the function at any other point by using the expansion of Colin MacLaurin. You don't need forever to understand Eternity – simply study a Moment; at a push any moment will do but I suggest that studying one of those Eternal Moments to which I referred last month will be more productive than studying a more humdrum moment.

Determinism

Sometimes when I succeed in explaining to my students my point that the future motion of a swing can be determined by studying in detail just one point in its swing, I am asked whether I believe that the future course of the universe, and our own destinies as a part of that universe, are predetermined by the present situation (in turn predetermined by past events). I am always pleased when that question is asked because it allows me to digress (I love digressions) into the nature of regularity (smoothness). I reply that I do not believe that the universe acts as if it is a regular (smooth) multi-variate function. I believe that you and I have a duty to 'push the swing' in the right direction and change the course of future events. I believe in mind-over-matter, not as a supernatural phenomenon but as an everyday occurrence to which we all contribute when we exercise our free will.

So that you know that I am aware of them let me mention briefly the butterfly effect, chaos theory, Werner Heisenburg' principle of indeterminacy and the paradox contained within the story of Erwin Schrödinger's cat. All these convince me that disbelief in determinism is rational though I accept that they do not prove the existence of free will.

Finally

(a) Please send a disc; (b) My address is that of Abacus Training; (c) Many thanks for all your letters.