Gerald's Column
by Gerald Fitton

It has been hard for me to resist commenting on the !CCShared contribution in last month's Archive but, apart from an "I told you so!", I shall resist the temptation and concentrate on other matters.

This month my major topic is recursive custom functions. Before you turn to someone else's column let me say that I think you'll understand what I've written and, in order to motive you to read what I have to say, I shall use as my example something which seems to be highly popular, a discussion of mathematical calculations relevant to the National Lottery.

First, three other small items.

Fireworkz Pro

By the time you read this, version 1.21 of Fireworkz Pro will be available (I don't have a copy yet – it's in the post – but I have been assured that the dateline of the new version is 27th April 1995). If you want an upgrade then you must send both your program and examples disc to Colton Software in a suitable padded envelope. It will help them if you enclose a self addressed sticky label. If you live in the UK then return postage will be appreciated and help speed up their service to you.

When I get my copy I'll let you know what differences there are between V 1.21 and V 1.20.

PC Viruses on the Archimedes

I have received many interesting letters from people who have contracted an Archimedes virus. Almost without exception they say that they thought it wouldn't happen to them – until it did. They echo my comment that it is a traumatic experience and they hope that it will never happen to them again. As a tutor and student counsellor at the college where I work I understand why writing to me and telling me of such traumas does help the writer to work it out of their system. Let me thank you for your letters and allow me to express my most sincere sympathy to you.

Nearly all of these letters end with the request that I should warn Archive readers to be careful so that the trauma which my correspondents have experienced should not be experienced by anyone else. Personally, I wish that some of these virus writers would stop thinking how clever they are and think about the heartache they are causing. They are abusing the talent they've been given.

I received a different sort of Virus letter (via the Archive office) from John Laski. He reports that he has discovered a PC type of virus in the MS DOS disk partition data of a friend's Archimedes. He asks if such a virus can get into parts of the disk where the "ARM expects partition data". Has anyone any answers or comment?

PipeDream row no. on the Risc PC

David Lenthall amongst others has written to me about the failure of PipeDream to display row numbers correctly on the Risc PC. Perhaps, if you have written to Colton Software about this problem you would let me know their reaction – after all, as many of you have pointed out to me, Colton Software are still selling PipeDream. Is the problem going to be fixed by Acorn in a future release of RISC OS or is it up to Colton Software to fix it?

I have been struggling to find a 'work around'. The best I can come up with is to insert an additional column before the first column so that it becomes a new column A. The formula 'row' will return the row number (eg 1234 in slot A1234). Enter 'row' in A1 replicate it to the bottom of the document. This process can be executed as a command file. I include one on the Archive monthly disc.

The Lottery

This month many of the letters I have received ask me if I can provide a spreadsheet application which will improve the user's chances of winning the lottery! Well, as a statistician I have to believe that the numbers which are selected by the lottery machine are random numbers. One of the properties of random numbers is that all methods of prediction fail. To put it as simply as I can; I can't help you to win the lottery!

However, what I can do is to help you to understand how to calculate your chances of winning and I'll comment on whether it is a worth while gamble! On the way there I shall introduce you to the joys of using recursive custom functions!

Some overseas subscribers to Archive may not know the details of our new National Lottery so I'll give a brief description of how it works. To participate you select six numbers from the numbers 1 to 49 (so there are forty nine different numbers to choose from). You can use each number only once in your selection of six. Unlike the typical office sweepstake, many people can choose the same set of six numbers as you.

You pay £1.00 for each set of six numbers you choose. Each Saturday evening at about 8 o'clock six numbers are selected at random from the forty nine using a glorified bingo machine full of numbered ping pong balls. A seventh ball is selected and this is called the bonus ball (more of which later). If your set of six balls is the same as those selected by the machine then you've won the Jackpot. If more than one person wins the Jackpot then it is shared out amongst the winners. There are also consolation prizes; for example if your set of six includes three of the six winning numbers then you win a fixed prize of £10.00.

The binomial coefficients

The number of possible sets of six numbers drawn from forty nine different numbers requires the evaluation of a well known mathematical function which generates what are called the binomial coefficients. Because the modern notation for this function is a little difficult (but not impossible) to include many times in a word processed document I shall call it by its old fashioned name, the nCr function.

This function has two parameters which are n, the number of numbers from which you can make your selection (in this case 49) and r, the number of numbers chosen (in this case 6). What we need to evaluate in this case is nCr(n,r) where n = 49 and r = 6.

The value of nCr(49,6) is given by: (49*48*47*46*45*44)/(6*5*4*3*2*1). You will notice that there are r (ie 6) numbers on the top and r numbers on the bottom of the fraction. The numbers on the top start at n (ie 49) and work downwards; those on the bottom start at r (ie 6) and work downwards.

Recurrence relationships

If you look at the expression for nCr(49,6) you will see that nCr(49,6) = (49/6)nCr(48,5). In other words we can express the value of nCr(49,6) in terms of nCr(48,5). In the same way we could express nCr(48,5) as (48/5)nCr(47,4) and so on – but not forever! In mathematics (long before computers) this type of general statement is an example of a recurrence relationship. It is:

nCr(n,r) = (n/r)*nCr(n-1,r-1) for values of r>1.

When r = 1 we have a problem because 1 - 1 = 0 and dividing by 0 is one of those things that mathematicians (and computers) find difficult! When r = 1 the value of nCr((n,1) = n. This leads us to a mathematical statement which we can write in PipeDream or Fireworkz format as:

if(r>1,nCr(n,r) = (n/r)*nCr(n-1,r-1),n)

Using recursion

There is a lot of snobbery about recursion. It is often portrayed as a programming feature which can be used only by an expert. Don't believe it!

There are many useful mathematical functions (such as the nCr function above) which are capable of simple expression as a recurrence relationship but which, in explicit form, look overwhelmingly difficult to understand! In these cases using recursion makes the program easier to write and easier to understand. When a program is easy to understand then changing (improving or expanding) it is much easier. In those cases I recommend using recursion. Where the reasoning behind the use of recursion is obscure I would suggest that the writer is just showing off!

There is a drawback to using recursion. I have to confess that recursion often takes longer to evaluate than the explicit (but more complicated) version.

The recursive function

In the next paragraph you will find the core of a PipeDream or Fireworkz custom function which will calculate these binomial coefficients. Usually I add much more comment to my custom functions to make them more readable.

The version on the Archive monthly disk includes the extra comment as well as lines which demonstrate the working of the custom function in detail.

...function(“binomial”,“n:number”,“r:number”)
Declare the ‘name’ of the one local variable, “answer”, and the slot it uses
...set_name(“answer”,B5
Initialise the local variable
...set_value(answer,0)
The next line contains the recursive call to the “binomial” function
...set_value(answer, if(@r>1,@n/@r*binomial(@n-1,@r-1),@n))
Return the result
...result(answer)

The custom function is called “binomial”. The recursive call is made in the line which starts “...set_value(answer,if . . .)”. This line includes a call to the function “binomial”, that is to say the function ‘binomial“ calls itself! It is this feature of a function calling itself which makes it recursive.

There is a difference in the arguments of the original function and the arguments used in the recursive call. The original function calculates nCr(n,r) whereas the recursive call calculates nCr(n-1,r-1). In fact, after stripping out all the non essential parts of the custom function nCr, what you are left with is the recurrence relationship:

nCr(n,r) = (n/r)*nCr(n-1,r-1).

What happens when the recursive function is executed is that the early part of the function (down to the point of recursion) is executed repeatedly for reducing values of n and r. During this part of the recursive procedure the latter part of the recursive function is never executed.

Naturally we can’t keep going inwards for ever. When you set up a recursive procedure then, like everything in life, it’s important to know when to stop! In the case of nCr we have to stop the recursive procedure when the value of r = 1 because the next incarnation would lead us to try to divide by zero. We achieve this halt to the recursion by using an if(,,) function. The innermost incarnation of “binomial” does not call itself because the if(r>1,,) ceases to be true.

What happens after the innermost incarnation of binomial has been executed? If I haven’t lost you yet then you’ll probably realise that it is the innermost incarnation of “binomial” which reaches the ‘...result(answer)’ line before any of the ‘outer’ incarnations. This value is returned to the next incarnation (going outwards) which then completes its execution down to ‘...result(answer)’.

Having ‘entered’ the recursive procedure r times we must ‘unwind’ it by running the ‘exit’ part of the custom function r times. One at a time the values of ‘...result(answer)’ are returned until every incarnation has been executed. Finally the last value is returned to the main spreadsheet.

Back to the the lottery

The value of nCr(49,6) is 13 986 816. This means that there are just under 14 million different lines of six numbers available to you (and others) when you choose your six numbers from forty nine. Your chance of winning with just one line is about 14 million to 1 against!

The £10.00 prize

The number of ways of selecting three numbers from the winning six and a further three numbers from the loosing forty three is nCr(6,3)*nCr(43,3) which is 246 820.

DeRef(slotref)

Let me use this calculation to explain something that has puzzled many people using the same custom function twice within the same slot. What I have to say applies to all custom functions and not only recursive functions.

If you use the custom function as I've written it then you will find that instead of returning the value you want the custom function will return a much larger number. This larger number is nCr(43,3)*nCr(43,3). Somehow or other the spreadsheet has lost the value of nCr(6,3) and substituted nCr(43,3). I would classify this as a bug but it is an obscure one with an obscure work around. The work around is to change the last line of the custom function from '...result(answer)' to '...result(deref(answer))'. You will find some (very limited) information about the deref() function in the PipeDream and Fireworkz handbooks but they won't tell you that you should use deref() as a matter of course when using custom functions that might be repeated within the same slot.

I have included on the Archive monthly disc a much simpler example (a custom function which adds together a couple of numbers) which gives the incorrect answer when used twice within the same slot unless the deref() construct is used.

Four of the six winning numbers

This is evaluated as from nCr(6,4)*nCr(43,2). There are 13 545 of the (approximately) 14 million lines which include four of the six winning numbers.

Five of the six winning numbers

The formula this time is nCr(6,5)*nCr(42,1) and gives the answer 258.

Five plus bonus

There is one more combination which results in a prize, usually quite a good one.

To win this prize you must have five of the six winning numbers and your sixth ball must be the seventh ball drawn by the lottery machine; the last ball called the bonus ball. There are nCr(6,5) = 6 lines (out of the 14 million) which can win this prize.

Gerald's second rule of gambling

Now let me discuss whether it is worth while betting on the lottery.

For purposes of teaching subjects such as Management Decision Making (where the outcome of such management decisions is uncertain) I have invented a few rules of gambling. My second rule is "Don't gamble what you can't afford to lose".

This rule is not intended to be a rule which relates to individual morality – though there are moral overtones and, as a tutor and counsellor of young students, I do come into contact with all too many who are addicted to gambling. No! My second rule is one which should be applied when the outcome of a decision is uncertain. I usually start my lesson with a short discussion of gambling in general but quite quickly get around to a discussion of buying insurance. You may think that buying insurance has many things in common with betting on a horse in a race! I take great pleasure in pointing out that if you don't buy insurance then you are also taking a gamble; you are gambling your house against the possibility that it won't burn down!

I go on to introduce the concept of a 'fair bet'. A fair bet is one where the amount you can win multiplied by the probability of winning equals the stake. As an example, if you were to bet me 1 unit that a single throw of a dice will result in a six then, for a fair bet, I should offer you odds which will return 6 units to you if you win.

I continue my lecture by suggesting that there are situations where the application of my second rule implies that it would be wrong to bet with a 'bookie' who offers you a fair bet rather than one loaded in his favour!

If you don't insure your house then you are taking a gamble that most of you can't afford to lose! I point out that the application of my second rule to such an insurance situation requires that, as the person insured, you should place your 'bet' with an insurance company which doesn't offer you a fair bet – you should bet with one where the odds are loaded in the insurance company's favour (but not too much)! Why? I hesitate to use the word "win" in this context so I'll say that it's because you want the company to be there and to pay out when your number comes up!

Applied to management decisions the rule requires you to pass up on some bets where the odds are in your favour because you can't afford to lose. I think that Nick Leeson's recently publicised bet on the Singapore derivatives market was of that type. The odds were in his favour (he'd proved that in the past) but it was a bet he (or rather his employers) couldn't afford to lose.

A corollary of my second rule is that there are some occasions when you may bet a small amount (which you can afford to lose) even though you are not offered a fair bet because the consequence of winning is to change your life completely.

The Lottery is not a fair bet because the money returned in prizes is much less than the total stake money. Nevertheless you could convince yourself that it is OK to make the bet (but only if you can afford to lose the £1.00 stake) because the consequences of success would be to change your life.

Do I bet on the lottery?

No I don't. I am not sure of my reasons but, somewhere deep down I feel the National Lottery is immoral. I don't feel the same way about my premium bonds so maybe I'm being inconsistent. Honestly I don't know why I feel different about the two different manifestations of gambling. I don't think that it's anything to do with the charitable aspect of the lottery money however, I do prefer to support a local hospice with my charitable donations rather than let someone else decide where the money should go. This may be because, at the local hospice, the terminal care given to a 21 year old relative of mine was quite wonderful.

To summarise, my second law of gambling will allow you to make an unfair bet but you should work out how unfair you think it is.

Marginal returns

In my decision making course I also discuss the concept of the marginal return as applied to big gambles. The following is my hypothetical scenario. I am a multi millionaire. I suggest to you that you bet me one penny against my million pounds on the toss of an unbiassed coin. You take the bet because it is an unfair bet loaded in your favour. You win. Your life will be different from here on! Now I suggest that we bet again but this time I bet ten million pounds and you must put up the whole of your one million (you can't put up only some of it). It is still an unfair bet loaded in your favour but I ask you, would you take the bet?

I have a rule about this situation too but it is rather complicated so I won't quote it. What I would ask you is "Would you prefer the lottery to have more Jackpot winners of, say £1M instead of the multi million winners which the lottery currently creates?

Finally

At least so far as the lottery is concerned, although there is no way in which I (or anyone else) can predict the numbers which will be generated by a random number generator there is something which might be useful to lottery punters. That is an application which looks at the number of people who win with different number combinations. There is no doubt that some numbers are more popular than others and that, when these popular numbers come up then the prizes are smaller than average. The converse of this is that if you bet regularly on unpopular numbers then, when your numbers come up, even for one of the minor prizes, you will receive more than that you expect if you based your calculations on the odds for a fair bet.

If you have a database which includes both the list of winning numbers and the number of people who have won the different minor prizes then I have a lot of correspondents who would like a copy!

In conclusion

Although letters addressed to me via the Archive office will get hear eventually it is usually much quicker to send them to me direct. My address is that of Abacus Training. I prefer disk copies of correspondence with a short hand written note outlining what is on the disk, a self addressed sticky label and, if possible, return postage.